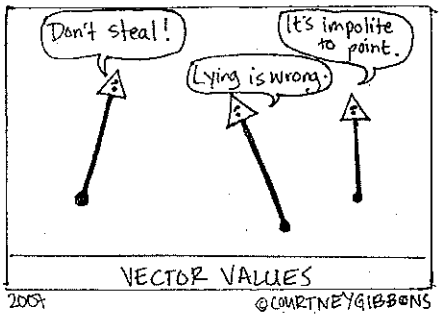
KEY

Math 225: Quiz the Fourth
September 30, 2011

This quiz is closed book and closed notes. Please justify all of your answers. You have 40 minutes.



1. Identify the following surfaces, *and* answer the related questions.

   (a) $2x^2 + 18y^2 + 32z^2 = 288$ (Give maximum values for x, y, and z).

   $\frac{x^2}{144} + \frac{y^2}{16} + \frac{z^2}{9} = 1$ ellipsoid

   $x \le 12 \quad y \le 4 \quad z \le 3.$

   (b) $2x^2 - 18y^2 + 32z^2 = 288$ (Give values of y which don't appear on this graph)

   $\frac{x^2}{144} + \frac{z^2}{9} = \frac{y^2}{16} + 1$ hyp. of 1 sheet

   → all values of y.

   6

   (c) $2x^2 - 18y^2 + 32z^2 = -288$ (Give values of y which don't appear on this graph)

   $\frac{x^2}{144} + \frac{z^2}{9} + 1 = \frac{y^2}{16}$ ~~$y \ge 4$ or y~~ $-4 \le y \le 4$ does not appear.

   hyp. of 2 sheets.

   (d) $2x^2 - 18y + 32z^2 = 0$

   $2x^2 + 32z^2 = 18y$ paraboloid

2. (a) Find the angle of intersection of the two planes $x + 3y + z = 5$ and $2x + 4y - z = 2$. (You may leave your answer as an arccosine.)

$\vec{n}_1 = \langle 1, 3, 1 \rangle$

$\vec{n}_2 = \langle 2, 4, -1 \rangle$

Angle

$\vec{n}_1 \cdot \vec{n}_2 = 2 + 12 - 1 = 13$

(3) $\theta = \arccos\left(\frac{13}{\sqrt{11} \cdot \sqrt{21}}\right)$

(b) Find the line of intersection of the two planes $x + 3y + z = 5$ and $2x + 4y - z = 2$

line: direction vector

$\vec{n}_1 \times \vec{n}_2 = \langle 1, 3, 1 \rangle \times \langle 2, 4, -1 \rangle = \langle -7, +3, -2 \rangle$

(4)

point: set $x = 0$ and solve

$3y + z = 5$

$4y - z = 2$

$\rightarrow y = 1, z = 2$

point $= (0, 1, 2)$

line: $\langle 0, 1, 2 \rangle + t\langle -7, 3, -2 \rangle = \vec{l}(t)$

(c) Find the equation of the plane that contains this line and the origin.

Two vectors in the plane

(4) $\langle 0, 1, 2 \rangle = \vec{v}_1$

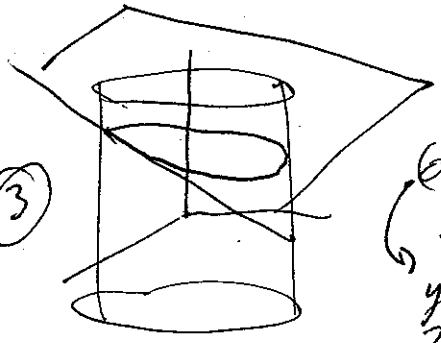
and $\langle -7, 3, -2 \rangle = \vec{v}_2$

$\vec{v}_1 \times \vec{v}_2 = \langle -8, -14, 7 \rangle$

plane: $-8x - 14y + 7z = 0$

cylinder plane

3. Parametrize and describe the curve of intersection between $x^2 + z^2 = 1$ and $x + y + z = 4$.



(3)

intersection of a cylinder.

(other answers possible...)

$x = \cos t$

$y = 4 - \cos t - \sin t$.

$z = \sin t$

4. (a) Find the equation of the tangent line to $\mathbf{r}(t) = \langle t, \sin(t), t^2 \rangle$ when $t = \pi$.

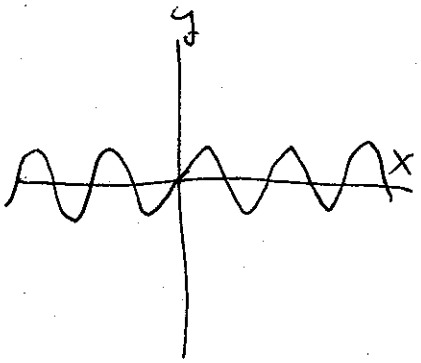
$\vec{r}(\pi) = \langle \pi, 0, \pi^2 \rangle$

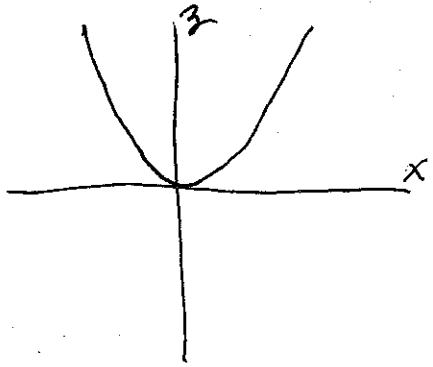
$\vec{r}\,'(t) = \langle 1, \cos t, 2t \rangle$

$\vec{r}\,'(\pi) = \langle 1, -1, 2\pi \rangle$

(3) line $\vec{\ell}(t) = \langle \pi, 0, \pi^2 \rangle + t\langle 1, -1, 2\pi \rangle$

(b) Draw the projections of $\mathbf{r}(t)$ onto the xy-plane and onto the xz-plane.





(2) $y = \sin x$ $z = x^2$

5. Extra Credit:

(a) What is your birthday? (Month and Day only. No years. Please.)

Nov. 19

(b) Of the 57 of us in the two classes, what is the approximate probability that at least 2 of us have the same birthday?

i. 16 % ii. 32 % iii. 70 % iv. 99 %