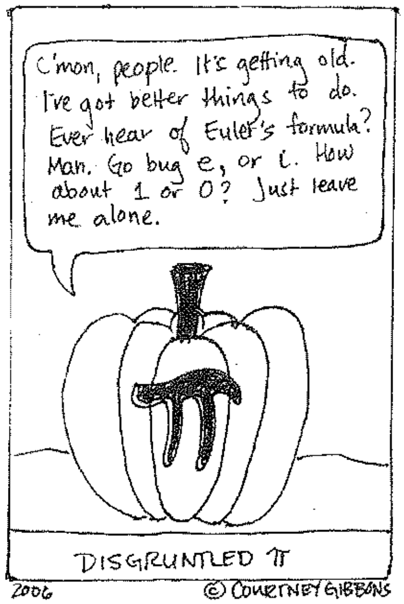
KEY

Math 225: Quiz the Fifth

September 30, 2011

This quiz is closed book and closed notes. Please justify all of your answers. You have 40 minutes.



1. Let $\mathbf{r}(t) = \langle t^2, e^t, t^3 \rangle$ describe the motion of a particle in space.

   (a) Express the velocity and acceleration of this particle as vector valued functions.

$$\vec{r}'(t) = \vec{v}(t) = \langle 2t, e^t, 3t^2 \rangle$$

$$\vec{r}''(t) = \vec{a}(t) = \langle 2, e^t, 6t \rangle$$

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   (b) Find the tangential and normal components of the acceleration vector. (Note: You need not compute $\mathbf{T}$ and $\mathbf{N}$.

$$a_T = \frac{\vec{r}' \cdot \vec{r}''}{|\vec{r}'|} = \frac{4t + e^{2t} + 18t^3}{\sqrt{4t^2 + e^{2t} + 9t^4}}$$

$$a_N = \frac{|\vec{r}' \times \vec{r}''|}{|\vec{r}'|} = \frac{|\langle e^t(6t-3t^2), -6t^2, (2t-2)e^t \rangle|}{\sqrt{4t^2 + e^{2t} + 9t^4}}$$

$$= \frac{\sqrt{e^{2t}(6t-3t^2) + 36t^4 + e^{2t}(2t-2)^2}}{\sqrt{4t^2 + e^{2t} + 9t^4}}$$

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2. Let $f(x, y) = xy$.

(a) Draw level curves for $f(x, y) = -2, -1, 0, 1, 2$.

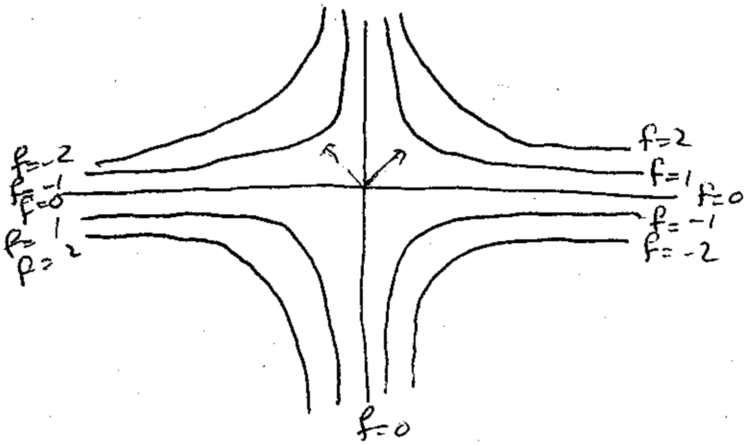
$xy = -2 \Rightarrow y = -2/x$

$xy = -1 \Rightarrow y = -1/x$

$xy = 0 \Rightarrow x = 0$ or $y = 0$

$xy = 1 \Rightarrow y = 1/x$

$xy = 2 \Rightarrow y = 2/x$



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(b) As we move from $(0, 0)$ to $(1, 1)$, does our 'elevation' increase or decrease?

our elevation increases as z is increasing

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(c) As we move from $(0, 0)$ to $(-1, 1)$, does our 'elevation' increase or decrease?

our elevation decrease as z is decreasing

(d) Does (0,0) sit at a 'valley', a 'peak', or neither? Explain.

(0,0) is neither a peak nor a valley → increase in all directions

↳ decrease in all directions

it is more of a "saddle" point.

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3. Calculate

$$\lim_{(x,y)\to(0,0)} \frac{x+y}{\sqrt{x^2+y^2}}$$

or show that it does not exist.

If $x=0$, $\lim_{y\to 0} \frac{y}{\sqrt{y^2}} = \lim_{y\to 0} \frac{y}{y} = 1$ > so limit DNE.

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But if $x=y$ $\lim_{y\to 0} \frac{2y}{\sqrt{y^2+y^2}} = \frac{2y}{\sqrt{2y^2}} = \frac{2}{\sqrt{2}} \neq 1$

4. Let $f(x,y) = x^2y + e^x + \cos(y)$

(a) Calculate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

$f_x = 2xy + e^x$

$f_y = x^2 + (-\sin y) = x^2 - \sin y$

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(b) Calculate all second-order partial derivatives, and verify that Clairaut's Theorem holds.

$f_{xx} = 2y + e^x$

$f_{xy} = 2x$

$f_{yx} = 2x$ → Clairaut's Theorem holds as $f_{xy} = f_{yx}$

$f_{yy} = -\cos y$

5. (Bonus) Pretend to toss a coin five times and write down your results as a string of heads and tails.

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