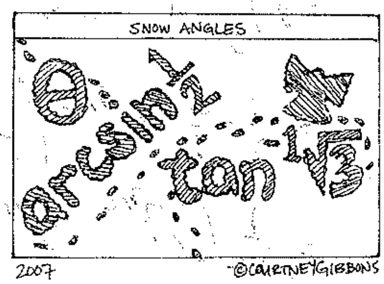
KEY:

I will take the final (circle one) WED AM FRI PM

Math 225: Quiz the Last

December 9, 2011

This quiz is closed book and closed notes. Please justify all of your answers. You have 40 minutes.



A word of advice: Go once through the test and set up your integrals, then back through and evaluate them.

1. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle \overset{P}{2xy + 2x}, \overset{Q}{x^2 + y^2} \rangle$ and C is the line segment from $(-1,1)$ to $(2,3)$.

Check: Is $\vec{F}$ conservative

$\frac{\partial Q}{\partial x} = 2x \qquad \frac{\partial P}{\partial y} = 2x \qquad$ yes!

Find f s.t. $\langle P, Q \rangle = \vec{\nabla} f$

$f_x = 2xy + 2x \qquad f = x^2 y + x^2 + g(y)$

$\downarrow \partial/\partial y$

$f_y = x^2 + y^2 \rightarrow f_y = x^2 + 0 + g'(y)$

$g'(y) = y^2 \qquad g(y) = y^3/3$

$\rightarrow f = x^2 y + x^2 + \frac{y^3}{3} \Big|_{(-1,1)}^{(2,3)} = 4\cdot 3 + 4 + 9 - (1 + 1 + \tfrac{1}{3})$

$25 - (2\tfrac{1}{3}) = \boxed{22\,^2/_3}$

2. Evaluate $\int_C x\, ds$ where C is the parabolic segment of $y = x^2 + 1$ from $(1, 2)$ to $(3, 10)$.

$$y = x^2 + 1$$

$$\rightarrow \quad x = t \qquad ds = \sqrt{(x')^2 + (y')^2}$$

$$y = t^2 + 1 \qquad = \sqrt{1 + (2t)^2}$$

$$1 \le t \le 3$$

$$\int_1^3 t\sqrt{1+4t^2}\, dt \quad = \frac{1}{8}\int_5^{37} u^{1/2}\, du$$

$u = 1 + 4t^2 \quad$ ul: 37

$du = 8t\, dt \quad$ ll: 5

$$= \frac{1}{8}\cdot\frac{2}{3}\, u^{3/2}\Big|_5^{37}$$

$$= \frac{1}{12}\left(37^{3/2} - 5^{3/2}\right)$$

3. Evaluate $\iiint_E z\, dV$, where E is bound by $x^2 + y^2 + z^2 = 9$ in the first octant.

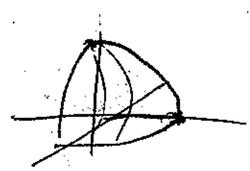
sphericals!

$$z = \rho\cos\phi$$

$$0 \le \rho \le 3$$

$$0 \le \theta \le \pi/2$$

$$0 \le \phi \le \pi/2$$



$$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^3 \rho\cos\phi\, \rho^2\sin\phi\, d\rho\, d\theta\, d\phi$$

$$= \int_0^{\pi/2}\int_0^{\pi/2}\int_0^3 \rho^3\cos\phi\sin\phi\, d\rho\, d\theta\, d\phi$$

$$= \int_0^{\pi/2}\int_0^{\pi/2} \frac{\rho^4}{4}\Big|_0^3 \cos\phi\sin\phi\, d\theta\, d\phi$$

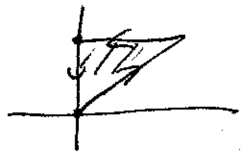
$$= \int_0^{\pi/2}\int_0^{\pi/2} \frac{81}{4}\cos\phi\sin\phi\, d\theta\, d\phi = \int_0^{\pi/2} \frac{81\pi}{8}\cos\phi\sin\phi\, d\phi \quad (u\text{-sub})$$

$$= \frac{81\pi}{16}\sin^2\phi\Big|_0^{\pi/2} = \frac{81\pi}{16}.$$

4. Evaluate

$$\oint_C \overbrace{(x^2 \ln x + 2y)}^{P}\,dx + \overbrace{(2x + \arcsin y)}^{Q}\,dy$$

where C is the triangle with vertices $(0,0)$, $(1,2)$, and $(0,2)$ traversed counterclockwise.



Is $\vec{F}$ conservative?

$\frac{\partial Q}{\partial x} = 2 \qquad \frac{\partial P}{\partial y} = 2 \qquad$ Yes!

Moreover ... C is Closed $\quad$ YES!

so $\oint_C \vec{F} \cdot d\vec{r} = \boxed{0}$

5. Evaluate

$$\oint_C (3x^2 y + y^3)\,dx + (y^2)\,dy,$$

where C is the unit circle centered at the origin, starting and ending at (1,0).

Is $\vec{F}$ conservative

$\frac{\partial Q}{\partial x} = 0 \qquad \frac{\partial P}{\partial y} = 3x^2 + 3y^2 \qquad$ no ...

But! Green's Thm applies

so $\oint_C \vec{F} \cdot d\vec{r} = \iint_R \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\, dA$

$= \iint_R (-3x^2 - 3y^2)\, dA = \int_0^{2\pi}\int_0^1 -3r^2\, r\, dr\, d\theta$

$= \int_0^{2\pi}\int_0^1 -3r^3\, dr\, d\theta$

$= \int_0^{2\pi} -\frac{3}{4}\, d\theta = -\frac{6\pi}{4} = \boxed{-\frac{3\pi}{2}}$

Bonus: Choose 0.5, 1, or 1.5 points extra credit. Those choosing the *least* popular option will receive the EC.