Math 225: Exam One Review
Fall 2017

These problems are largely culled from last year's first midterm. Note that they do not represent the full spectrum of topics covered in class. For that, please consult your class notes, the course website, and past WebWork assignments. We will go over these in class on Wednesday, October 11.

1. (a) Eliminate the parameter to graph the parametric curve $x = 2t + 1, y = t$.
   (b) Use your graph from part (a) to graph the parametric curve for $x = t^2 + 1, y = \dfrac{t^2}{2}$.

2. Consider the parametric equations

$$x = t^2 - t, y = 2t - t^2$$

   (a) At what point does this curve have a vertical tangent?
   (b) For which values of t is $x \leq 0$ and $y \geq 0$?
   (c) Use these values to find the area bound by the curve and the y-axis in the *second* quadrant.
   (d) Give an equation of a hyperboloid of two sheets that has symmetry about the y-axis which goes through the points $(0, 3, 0)$ and $(0, -3, 0)$.

3. Let $A = (3, 1, 2)$, $B = (7, 3, 6)$ and $P = (x, y, z)$.
   (a) Find the distance from A to B.
   (b) Write an equation to express

$$\overrightarrow{AP} \perp \overrightarrow{BP}.$$

   (c) Simplify your equation to the formula for a sphere. What are the center and radius of the sphere relative to A and B?

4. Suppose that $\mathbf{a} \perp \mathbf{b}$. Prove that $|\mathbf{a} + \mathbf{b}| = |\mathbf{a} - \mathbf{b}|$. (Draw a picture if it helps).

5. Find the equation of the line through the point (2,1,4) that is perpendicular to the plane $4x - y + 3z = 7$.

6. Find the plane of intersection of the lines $\langle 2 + t, 4t, 1 + 3t\rangle$ and $\langle -1 - 2s, 6 + s, 10 + 3s\rangle$.

7. In this exercise, we develop the formula for the distance between two parallel lines.
   (a) The figure on the board each with direction vector $\mathbf{v}$ starting at a point. We connect the starting points with a vector $\mathbf{b}$. Also, we label the length that will be the distance d between the lines at the head of vector $\mathbf{b}$.
   (b) Consider the angle θ between $\mathbf{b}$ and $\mathbf{v}$. We calculate $\sin(\theta)$ two ways...
      i. ...as a ratio involving the distance between the lines.
      ii. ...as a result of the cross product of $\mathbf{b}$ and $\mathbf{v}$.

(c) Now, set these calculations equal and simplify for an elegant formula for the distance d between the two lines.

(d) Use your formula to find the distance between $\langle 2+3t, 5t, -1+t\rangle$ and $\langle 1+3t, 4+5t, 7+t\rangle$.

8. Let $\mathbf{r}(t) = \langle \sin(2t), e^t + 2, t^3 + t + 3\rangle$

   (a) Find a unit vector tangent to $\mathbf{r}(t)$ when $t = 0$.

   (b) Find the tangent line to $\mathbf{r}(t)$ when $t = 0$.

   (c) If $\mathbf{r}(t)$ is the curve of an object in motion, is that object ever at 'rest'? Explain.

9. Suppose that $\mathbf{r}(t) = \langle t^2, \frac{2}{3}t^3, t\rangle$.

   (a) Find $\mathbf{T}(1)$

   (b) If $\mathbf{N}(1) = \langle \frac{-1}{3}, \frac{2}{3}, \frac{-2}{3}\rangle$, find $\mathbf{B}(1)$.

   (c) Calculuate $a_{\mathbf{T}}$ and $a_{\mathbf{N}}$ at $t = 1$

10. Consider the function $f(x, y) = \dfrac{1}{\sqrt{x^2 + y^2 - 1}}$

   (a) Find f_x, f_y and or this function.

   (b) Find and sketch the domain for this function.

   (c) Draw level curves for this surface corresponding to $k = \frac{1}{4}, \frac{1}{2}, 1, 2, 4$.

   (d) What 'geologic' structure does the graph of f resemble?