Math 225: Review for Exam the Second
Fall 2017

This is a representative (though not exhaustive!) list of the types of problems you are likely to encounter on Wednesday's Exam.

1. (a) Find the equation of the tangent plane to the function $f(x,y) = \dfrac{x}{x+y}$ at the point (2,1).
   (b) Find $D_{\mathbf{u}}(f)$ as we move from (2,1) in the direction of $\mathbf{v} = \langle 3, 4 \rangle$.
2. (a) Find the equation of the tangent plane to $f(x,y) = \ln(x-2y)$ at the point $(3,1)$.
   (b) At the point $(3,1)$, in what direction should you move to increase the fastest?
3. Find the tangent plane to the equation $x^2 + 3xy + 2xyz = 11$ at the point (1,2,1). Also, find $\frac{\partial x}{\partial y}$ at this point.
4. Find the maximum and minimum value of $f(x,y,z) = x^2y^2z^2$ subject to the constraint $x^2 + y^2 + z^2 = 1$.
5. A cylindrical can is to have surface area 54π square inches. Find the dimensions that maximize the volume of the can. You may use any method that you wish.
6. Consider the function $f(x,y) = x^2 + kxy + y^2$, where k is a constant.
   (a) Show that f has a critical point at $(0,0)$ regardless of the choice of k.
   (b) For which values of k is $(0,0)$ a local minimum?
   (c) For which values of k is $(0,0)$ a local maximum?
   (d) For which values of k is $(0,0)$ a saddle point?
   (e) For which values of k does the discriminant test require more investigation?
   (f) Investigate f at these values of k and classify (hint: f factors nicely in these cases).
7. Find a function $f(x,y)$ such that $f_x = 2x + 3y^2$ and $f_y = 6xy + 7$, or explain why none exists.
8. Find the average value of $f(x,y) = x^2 + 2y$ on the rectangle $R = [1,2] \times [3,6]$
9. Find
   $$\iint_R x\cos(3xy)\, dA$$
   where $R = [0,1] \times [0,\pi]$.
10. Find
   $$\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\, dx\, dy.$$