MIDTERM #2 REVIEW · ODDS 11/06/17

1. $f(x,y) = \frac{x}{x+y}$ @ $(2,1)$ T plane: $z = \underline{f(2,1)} + \underline{f_x}(x-2) + \underline{f_y}(y-1)$

$f(2,1) = \frac{2}{2+1} = \frac{2}{3}$

$f_x = \frac{(x+y)(1) - x(1)}{(x+y)^2} = \frac{y}{(x+y)^2}\Big|_{(2,1)} = \frac{1}{9}$

$f_y = \frac{d}{dy}\, x(x+y)^{-1} = x(-(x+y)^{-2}) = \frac{-x}{(x+y)^2} = -\frac{2}{9}$

a. $\boxed{z = \frac{2}{3} + \frac{1}{9}(x-2) - \frac{2}{9}(y-1)}$

$D_{\vec{u}}f$ $(2,1)$ in the direction of $\vec{v} = \langle 3,4 \rangle$

$D_{\vec{u}}f = \vec{\nabla} f \cdot \vec{u}$ $\qquad \vec{u} = \langle \frac{3}{5}, \frac{4}{5} \rangle$

$= \langle \frac{1}{9}, -\frac{2}{9} \rangle \cdot \langle \frac{3}{5}, \frac{4}{5} \rangle$

$= \frac{3-8}{45} =$ b. $\boxed{-\frac{1}{9}}$

3. Find the tangent plane to $x^2 + 3xy + 2xyz = 11$ @ $(1,2,1)$. Also find $\frac{dx}{dy}$ @ $(1,2,1)$

a. b.

tan plane: need $\vec{n}$ and a point

point: $(1,2,1)$

$\vec{n} = \vec{\nabla} F = \langle 2x + 3y + 2yz, 3x + 2xz, 2xy \rangle = \langle 12, 5, 4 \rangle$

plane: $\boxed{12(x-1) + 5(y-2) + 4(z-1) = 0}$

NOTE: this tangent plane is different because it is in implicit form

$\frac{dx}{dy} = -\frac{F_y}{F_x} = \boxed{-\frac{5}{12}}$] how much x will move if we tweek y by a bit → only when $F_y \neq 0$

5.

$SA = 64\pi = 2\pi r^2 + 2\pi rh$ find max volume

$V = \pi r^2 h$

$V_r = \lambda SA_r$: $2\pi rh = \lambda(4\pi r + 2\pi h) = \frac{r}{2}(4\pi r + 2\pi h)$

$V_h = \lambda SA_h$: $\pi r^2 = \lambda(2\pi r) \longrightarrow \lambda = \frac{\pi r^2}{2\pi r} = \frac{r}{2}$

$\cancel{64\pi} = \cancel{2\pi} r^2 + \cancel{2\pi} rh$

$64 = 2r^2 + 2r(2r)$

$64 = 2r^2 + 4r^2$

$64 = 6r^2$

$9 = r^2 \quad r = \pm 3$

only the positive makes sense

$\boxed{\begin{array}{l} r = 3 \\ h = 6 \end{array}}$

$\cancel{2\pi} rh = \frac{\cancel{4}^{2}\cancel{\pi} r^{\cancel{2}}}{\cancel{2}} + \frac{\cancel{2\pi} rh}{\cancel{2}}$

$2h = 2r + h$

$\underline{h = 2r}$

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7. Find a function $f(x,y)$ such that $f_x = 2x + 3y^2$ $f_y = 6xy + 7$, or explain why none exist

$$\checkmark f_{xy} = 6y \qquad \checkmark f_{yx} = 6y$$

$$f(x,y) = x^2 + 3xy^2 + g(y)$$

$$f_y = 6xy + g'(y) = 6xy + 7$$

$$g'(y) = 7$$

$$g(y) = 7y$$

$$\boxed{f(x,y) = x^2 + 3xy^2 + 7y + C}$$

9. $R = [0,1] \times [0,\pi]$ $\iint_R x \cos 3xy \, dA \rightarrow \int_0^1 \int_0^\pi \cancel{x} \cos(3xy) \, dy \, dx$ $u = 3xy$, $du = \cancel{3x} dy$

$$\int_0^1 \int_0^{3\pi x} \frac{1}{3} \cos(u) \, du \, dx$$ NOTE: change bounds in terms of "u"

$$\int_0^1 \frac{1}{3} \sin(u) \Big|_0^{3\pi x} dx = \frac{1}{3} \int_0^1 \sin(3\pi x) \, dx = \frac{-\cos 3\pi x}{3 \cdot 3\pi} \Big|_0^1$$

$$= \frac{-\cos(3\pi)}{9\pi} - \left(-\frac{\cos(0)}{9\pi}\right) = \frac{1}{9\pi} - \left(-\frac{1}{9\pi}\right) = \boxed{\frac{2}{9\pi}}$$

,1)

Mid Term Review

2)

$f(x,y) = \ln(x-2y)$ at $(3,1)$

a) $f(3,1) = \ln(3-2) = \ln(1) = 0$

$f_x = \frac{1}{x-2y} \cdot (1) \Big|_{(3,1)} = 1$

$f_y = \frac{1}{x-2y} \cdot (-2) \Big|_{(3,1)} = -2$

Tangent Plane:

$z = 0 + 1(x-3) - 2(y-1)$

b) $\vec{\nabla} f = \langle 1, -2 \rangle \leftarrow$ Direction of increase

$\langle -1, 2 \rangle \leftarrow$ Decrease fastest

Max slope : $\sqrt{5}$

$\langle 2, 1 \rangle$ or $\langle -2, -1 \rangle \leftarrow$ No change in f

4)

$f(x,y,z) = x^2y^2z^2$ subject to $x^2+y^2+z^2=1$

$$\left.\begin{aligned} f_x &= 2xy^2z^2 = 2x\lambda \Rightarrow 2x^2y^2z^2 = 2x^2\lambda \\ f_y &= 2x^2yz^2 = 2y\lambda \Rightarrow 2x^2y^2z^2 = 2y^2\lambda \\ f_z &= 2x^2y^2z = 2z\lambda \Rightarrow 2x^2y^2z^2 = 2z^2\lambda \end{aligned}\right\} \quad x^2 = y^2 = z^2$$

$x^2 = \frac{1}{3} \leftarrow x^2+x^2+x^2=1$

$x = \pm \frac{\sqrt{3}}{3} = y = z$

MAX $= \frac{1}{27}$

6]

$f(x,y) = x^2 + kxy + y^2$

$f_x = 2x + ky = 0$
$f_y = kx + 2y = 0$

(0,0) satisfies both equations so there is a critical point at (0,0)

$D = f_{xx} f_{yy} - (f_{xy})^2$
$= (2)(2) - k^2$
$= 4 - k^2$

> 0 max or (min) $-2 < k < 2$ (never)
< 0 saddle point $k > 2$ or $k < -2$
$= 0$:(

Further Investigation

$k = 2$
$x^2 + 2xy + y^2 = (x+y)^2$
$k = -2$
$x^2 - 2xy + y^2 = (x-y)^2$

$f(0,0) = 0$
minimum (of sorts because it is not the only one)

In both cases $f(x,y) \geq 0$

8]

Average value of: $f(x,y) = x^2 + 2y$ on $[1,2] \times [3,6]$

$\frac{1}{(6-3)(2-1)} \int_1^2 \int_3^6 x^2 + 2y \, dy \, dx$

$= \frac{1}{3} \int_1^2 x^2 y + y^2 \Big|_3^6 dx$

$= \frac{1}{3} \int_1^2 6x^2 + 36 - (3x^2 + 9) \, dx$

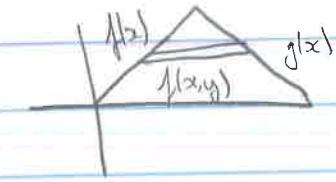
$= \frac{1}{3} \int_1^2 3x^2 + 27 \, dx = \frac{1}{3}(x^3 + 27x) \Big|_1^2$

$= \frac{1}{3}((8 + 54) - (1 + 27))$

$= \frac{1}{3}(34)$

$= \frac{34}{3}$

Average of f over R $\dfrac{\iint_R f(x,y)\,dA}{\iint_R 1\,dA}$



$\rightsquigarrow \int_a^b g(x) - f(x)\,dy$

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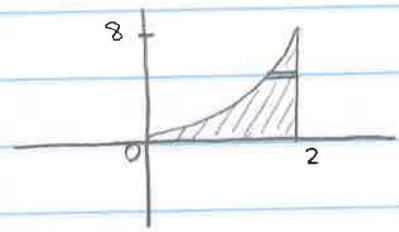
$$\int_0^8 \int_{3\sqrt{y}}^2 e^{x^4}\,dx\,dy$$



$x = 3\sqrt{y}$

$y = x^3$

$$= \int_0^2 \int_0^{x^3} e^{x^4}\,dy\,dx$$

$$= \int_0^2 y e^{x^4} \Big|_0^{x^3} dx$$

$$= \int_0^2 x^3 e^{x^4}\,dx$$

$$= \frac{1}{4} e^{x^4} \Big|_0^2$$

$$= \frac{1}{4}\left(e^{16} - 1\right)$$