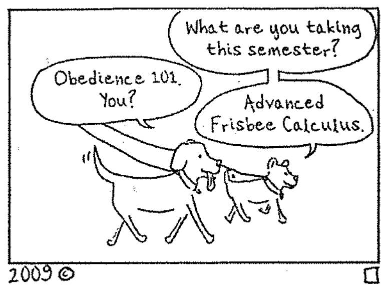
Key

Math 225: Quiz the First
September 8, 2017

This quiz is closed book and closed notes. You may use a calculator for arithmetic only on this quiz (i.e., no graphing and no calculus). Please justify all of your answers. You have the remainder of the period.



1. Give parametric equations for the following graphs

   (a) A line between the points (1,2) and (4, -3).

   $x = 1 + 3t$

   $y = 2 + (-5)t$

   (b) A circle of radius 2, centered at the origin, traced counterclockwise once, starting at the point (0,2).

   Either

   $x = 2\cos t$
   $y = 2\sin t$
   $\pi/2 \le t \le \frac{5\pi}{2}$

   OR

   $x = -2\sin t$
   $y = 2\cos t$
   $0 \le t \le 2\pi.$

2. Consider the curve $x(t) = 9 - t^2, y(t) = 3 - t$

(a) Find the *points* where the curve crosses the x- and y-axes (if any)

cross y-axis $x = 0$

$9 - t^2 = 0$

$t = \pm 3$

points $(0,0)$ $(0,6)$

cross x-axis $y = 0$

$3 - t = 0$

$t = 3$

point $(0,0)$

(b) Find the *points* where the curve has a horizontal or vertical tangent (if any).

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-1}{-2t} = \frac{1}{2t}$

$1 \neq 0$ no horiz tan

$2t = 0 \to t = 0$

$(9,3)$

vert tan @ $(9,3)$

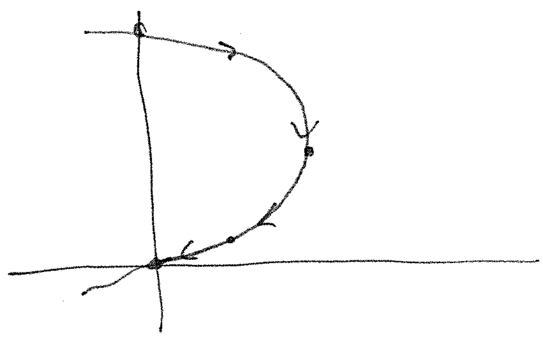
(c) Find the equation of the tangent line to the curve when $t = 1$. (Point-slope form is fine)

$t = 1$ point: $(9 - 1^2, 3 - 1) = (8, 2)$

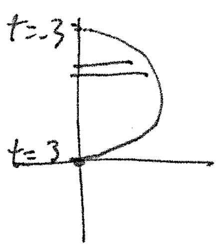
slope: $\left.\frac{1}{2t}\right|_{t=1} \to \frac{1}{2}$

line: $y - 2 = \frac{1}{2}(x - 8)$

(d) Draw a rough sketch of the curve, with attention to direction.



(e) Find the area bound by the curve and the y-axis.



$A = \int x\,dy = \int_3^{-3} (9 - t^2)(-1)\,dt$

$= \int_{-3}^{3} 9 - t^2 = 9t - \frac{t^3}{3}\Big|_{-3}^{3} = (27 - 9) - (-27 + 9)$

$= \underline{36}$

3. Find the center and the radius of the sphere $x^2 + y^2 + z^2 - 8x + 4z = 29$

$x^2 - 8x + 16 + y^2 + z^2 + 4z + 4 = 29 + 16 + 4$

$(x-4)^2 + y^2 + (z+2)^2 = 49$

c: $(4, 0, -2)$

Radius = 7.

A B C

4. Determine if the points (2, 4, 7), (0, 1, 9), and (6, 10, 3) are colinear.

$D_{AB} = \sqrt{(-2)^2+(-3)^2+(+2)^2} = \sqrt{17}$

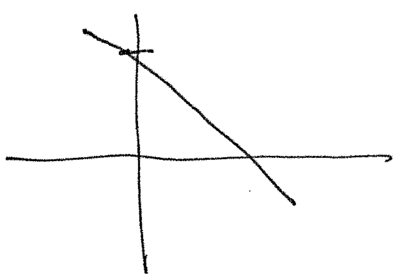
$D_{BC} = \sqrt{6^2+9^2+(-6)^2} = \sqrt{153} = 3\sqrt{17}$

$D_{AC} = \sqrt{4^2+6^2+(-4)^2} = \sqrt{68} = 2\sqrt{17}$

$D_{AB} + D_{AC} = D_{BC}$ so yes, they're colinear

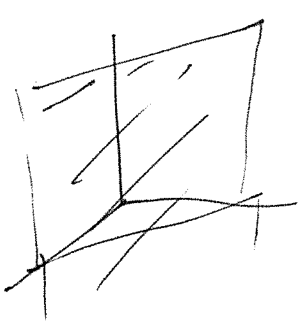
5. Describe (geometrically, and be descriptive) the set of points satisfying $x + y = 3$

(a) in $\mathbf{R}^2$



$x+y=3$ is a line with slope -1 and intercepts @ $(3,0)$ and $(0,3)$

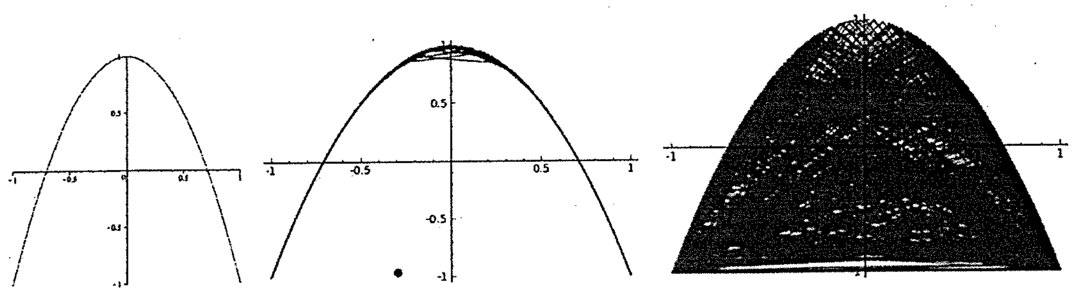
(b) in $\mathbf{R}^3$



$x+y=3$ is a vertical plane perpendicular to the xy plane (and parallel to the z-axis.)

(Note on being descriptive: If, for example, you were describing $(x = \cos(t), y = \sin(t))$ where $(0 \le t \le 2\pi)$, then a poor description would be 'It's a circle'. A good description would be 'It's a circle of radius 1, centered at the origin, moving counterclockwise from the point (1,0).

6. (Bonus) See the graphs below. They represent the graph of the polar equations $x = \sin(t)$ and $y = \cos(2t) = 1 - \sin^{(}t)$. The graph is of the form $y = 1 - 2x^2$, but restricted and periodic. The three graphs have increasing bounds on t. Why might we get different pictures for the graphs?



(The ranges on t are $(0, 2\pi)$, $(0, 100\pi)$, and $(0, 500\pi)$ The 'dot' in the second graph is merely a printing error).