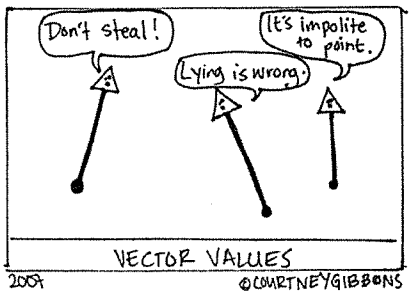
KEY

Math 225: Quiz the Second
September 15, 2017

This quiz is closed book and closed notes. Please justify all of your answers. You have until the end of the class period to finish.



1. Let $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ be vectors. For each quantity, state whether it is a vector, a scalar, or nonsense.

(a) $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$ = scalar

(b) $\mathbf{a} \times (\mathbf{b} \cdot \mathbf{c})$ = nonsense

(c) $(\mathbf{a} \cdot \mathbf{b})\mathbf{c} + (\mathbf{a} \cdot \mathbf{c})\mathbf{b}$ = vector

(d) $\mathbf{a}^2$ nonsense → no operation defined

2. Let $A = (0, 1, 2)$, $B = (2, 2, 4)$ and $C = (1, 6, 3)$

(a) Determine the vectors $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{BC}$

$\overrightarrow{AB} = \langle 2, 1, 2 \rangle$

$\overrightarrow{AC} = \langle 1, 5, 1 \rangle$

$\overrightarrow{BC} = \langle 1, 4, -1 \rangle$

(b) Find a vector parallel to $\overrightarrow{AB}$ that has length 5.

$|\overrightarrow{AB}| = \sqrt{4+1+4} = 3$

so if $\lambda = 5/3$, then $\vec{v} = \left\langle \frac{10}{3}, \frac{5}{3}, \frac{10}{3} \right\rangle$.

(c) Determine if $\triangle ABC$ is right, acute, or obtuse.

$\overrightarrow{AB} \cdot \overrightarrow{AC} > 0$

$\overrightarrow{AC} \cdot \overrightarrow{BC} > 0$

But $\overrightarrow{AB} \cdot \overrightarrow{BC} = -\overrightarrow{BA} \cdot \overrightarrow{BC} = 2 - 4 + 2 = 0$ so $\measuredangle B = 90°$

$\triangle ABC$ is right.

(d) Project $\overrightarrow{AC}$ onto $\overrightarrow{AB}$.

$\text{proj}_{\overrightarrow{AB}} \overrightarrow{AC} = \frac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{|\overrightarrow{AB}|^2} \overrightarrow{AB} = \frac{(2+5+2)}{(\sqrt{9})^2} \overrightarrow{AB} = \frac{9}{9} \overrightarrow{AB} = \overrightarrow{AB}$

$= \langle 1, 5, 1 \rangle$

(e) Find a vector orthogonal to both $\overrightarrow{BA}$ and $\overrightarrow{BC}$

$\overrightarrow{BA} = \langle -2, -1, -2 \rangle$

$\times \overrightarrow{BC} = \langle -1, 4, -1 \rangle$

$\langle 1 - 9, 2 - 2, -8 - 1 \rangle = \langle 9, 0, -9 \rangle$.

3. Find values s and t, if they exist, such that $s\langle 3,1,2\rangle + t\langle -1,1,2\rangle = \langle 3,5,10\rangle$.

$3s - t = 3$ $\}$ $4s = 8$, $s = 2$; $t = 3$

$s + t = 5$

$2s + 2t = 10$

So $2\langle 3,1,2\rangle + 3\langle -1,1,2\rangle = \langle 3,5,10\rangle$.

4. Find all values b such that $\langle 1, b, -1\rangle$ is orthogonal to $\langle -b, 2b, 3\rangle$.

$\langle 1, b, -1\rangle \cdot \langle -b, 2b, 3\rangle = 0$

$-b + 2b^2 - 3 = 0$

$2b^2 - b - 3 = 0$

$(2b - 3)(b + 1) = 0$

$b = 3/2, -1$

5. (a) Under what circumstances is $\mathbf{comp_b a} = \mathbf{comp_a b}$?

$\text{Comp}_{\vec{b}}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} \stackrel{?}{=} \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = \text{Comp}_{\vec{a}}\vec{b}$

When $|\vec{a}| = |\vec{b}|$ or when $\vec{a}\cdot\vec{b} = 0$

(b) Under what circumstances is $\mathbf{proj_b a} = \mathbf{proj_a b}$?

$\text{proj}_{\vec{b}}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\vec{a} = \text{proj}_{\vec{a}}\vec{b}$

When $\vec{a} = \vec{b}$ or $\vec{a}\cdot\vec{b} = 0$

6. (Bonus)

(a) You have a clock with 12 vectors, each pointing from the center to one of the numbers. What is the sum of these vectors?

(b) You have a clock with 11 vectors, each pointing from the center to a number, except one pointing to '4 o'clock'. What is the sum of these vectors?

(c) You have a clock with 11 vectors, each pointing from 6 o'clock to one of the other numbers. What is the sum of these vectors (remember to give direction and magnitude)?

a) $\vec{0}$

b) "10 o'clock"

c) straight up from 6 o'clock

6x $\overrightarrow{6 \text{ to } 12}$.