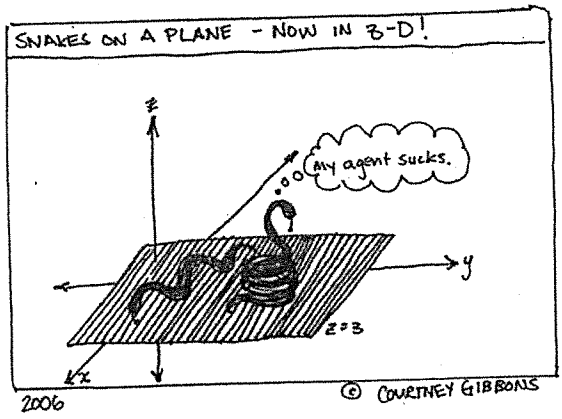
KEY

Math 225: Quiz the Third
September 22, 2017

This quiz is closed book and closed notes. Please justify all of your answers. You have until the end of the class period to finish.



1. Finish each sentence (in terms of vectors). All the action here is in R^3.

   (a) Two lines are parallel if....

   they have the "same" direction vector (parallel)

   (b) Two planes are parallel if...

   they have the "same" normal vector.

   (c) A line and a plane will *not* intersect if...

   the line's direction vector is perpendicular to the plane's normal vector.

3. Find the equation of the line through the points (3,1,2) and (4,0,-2). Give at least two forms for your equation(s).

$\vec{v} = \langle 1, -1, -4 \rangle$

$\vec{\ell}(t) = t\langle 1, -1, 4 \rangle + \langle 3, 1, 2 \rangle$

or $x = 3 + t$
$y = 1 - t$
$z = 2 + 4t$

4. Find the plane through the point (2,2,4) that is perpendicular to the line

$$\frac{x-1}{3} = 2 - y = 2z + 4$$

$\vec{n} = \langle 3, -1, \frac{1}{2} \rangle$

Point : $(2, 2, 4)$

plane : $3(x-2) - 1(y-2) + \frac{1}{2}(z-4) = 0$

or $6(x-2) - 2(y-2) + (z-4) = 0$

5. (a) Find the point of intersection of the lines $\mathbf{r}_1(t) = \langle 1+t, 1-t, 2t\rangle$ and $r_2(s) = \langle 2-s, s, 2\rangle$

$1+t = 2-s$

$1-t = s$

$2t = 2$

$t = 1$

Point: $(2, 0, 2)$

$1+1 = 2-s \rightarrow s = 0$

(b) Find the equation of the plane containing both of these lines.

$\vec{v}_1 = \langle 1, -1, 2\rangle$

$\vec{v}_2 = \langle -1, 1, 0\rangle$

$\vec{n} = \langle -2, -2, 0\rangle$

plane:

$-2(x-2) - 2(y-0) + 0(z-2) = 0$

or $-2x+4-2y = 0$

or $2x+2y = 4$, $x+y = 2$

(c) Find the distance from this plane to the point (5, 4, 3).

$$\text{Dist} = \frac{|ax_1+by_1+cz_1 = d|}{\sqrt{a^2+b^2+c^2}} \rightarrow \frac{|(1(5) + 1(4) + 0(3) - 2|}{\sqrt{1^2+1^2+0^2}}$$

$$= \frac{7}{\sqrt{2}}.$$

6. (Bonus) You may have half a point Extra Credit, or one-and-one-half points extra credit. Note: If more than 25% of you choose one-and-one-half, then no one gets anything.