


KEY

Math 225: Quiz the Fourth  
September 29, 2017

You have the remainder of the period to complete this quiz. You may use a calculator for arithmetic only.

<p>Dear students...</p> <p>I may or may not be in my office 3:30-5:30 today. In fact, I may be both in my office and not in my office while my office door is closed. You would not know the status of my whereabouts until you come and open my office door, at which point the status of my whereabouts may have changed depending on the way you have observed the interior of my office.</p> <p>Martin. Or not Martin. Or both.</p>	
<p><b>Schrödinger's Office Hours</b></p>	

1. Identify each surface below, and provide the requested information.

(a)  $4x^2 + 9y^2 + 36z^2 = 144$  (give also max/min  $x$ -values)

ellipsoid  $\rightarrow \frac{4x^2}{144} + \frac{9y^2}{144} + \frac{36z^2}{144} = 1$

$\hookrightarrow \frac{x^2}{36} + \frac{y^2}{16} + \frac{z^2}{4} = 1 \rightarrow -6 \leq x \leq 6$

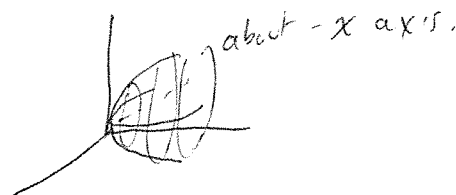
(b)  $x^2 - 9y^2 + 12z^2 = 1$  (give also the axis of symmetry)

$x^2 + 12z^2 = 1 + 9y^2$  hyperboloid of one sheet

$\hookrightarrow y$ -axis of symmetry

(c)  $x + y^2 + z^2 = 0$  (give a rough sketch of the graph)

$x = -y^2 - z^2 \leftarrow$  elliptical (circular) paraboloid

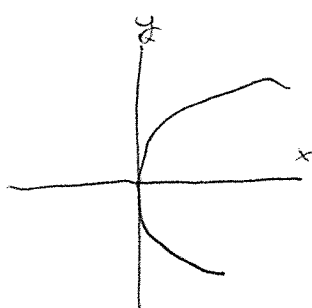


2. Consider the space curve  $\mathbf{r}(t) = \langle t^2, 2t, t^2 - 1 \rangle$ .

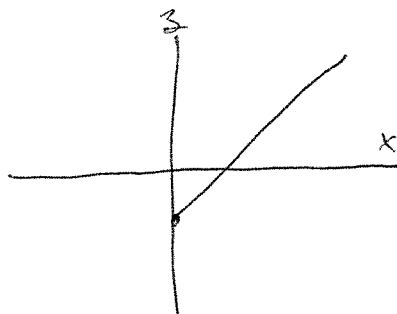
(a) What are the values that  $x$ ,  $y$ , and  $z$  take on?

$$\begin{aligned} x &= t^2 & x &\geq 0 \\ y &= 2t & y &\text{ is free} \\ z &= t^2 - 1 & z &\geq -1 \end{aligned}$$

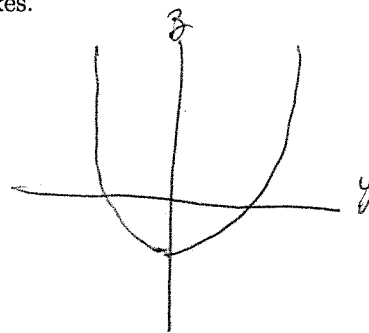
(b) Draw the views of the curve looking down the  $x$ ,  $y$  and  $z$ -axes.



$$x = \left(\frac{y}{2}\right)^2$$



$$y = x - 1 \text{ for pos. } x \text{ values}$$



$$z = \left(\frac{y}{2}\right)^2 - 1$$

(c) Find the equation of the tangent line to  $\mathbf{r}(t)$  when  $t = 3$ .

$$\vec{r}(3) = \langle 9, 6, 8 \rangle$$

$$\vec{r}'(3) = \langle 2t, 2, 2t \rangle \Big|_{t=3} \rightarrow \langle 6, 2, 6 \rangle$$

$$\text{line: } \vec{l}(t) = \langle 9, 6, 8 \rangle + t \langle 6, 2, 6 \rangle$$

3. Suppose that a particle in motion satisfies  $\mathbf{v}(t) = \langle 3t, \cos(t), \sqrt{t+1} \rangle$  for  $t \geq -1$  (where  $\mathbf{v}(t)$  is the velocity vector).

(a) Find the acceleration  $\mathbf{a}(t)$ .

$$\vec{a}(t) = \vec{v}'(t) = \langle 3, -\sin(t), \frac{1}{2\sqrt{t+1}} \rangle$$

(b) Find the position,  $\mathbf{s}(t)$ , if  $\mathbf{s}(0) = \langle 1, 2, 1 \rangle$

$$\begin{aligned} \mathbf{s}(t) &= \int_0^t \vec{v}(r) dr + \langle 1, 2, 1 \rangle \\ &= \left\langle \frac{3}{2}r^2, \sin r, \frac{2}{3}(r+1)^{3/2} \right\rangle \Big|_0^t + \langle 1, 2, 1 \rangle \\ &= \left\langle \frac{3}{2}t^2, \sin t, \frac{2}{3}(t+1)^{3/2} \right\rangle - \left\langle 0, 0, \frac{2}{3} \right\rangle + \langle 1, 2, 1 \rangle \\ &= \left\langle \frac{3}{2}t^2 + 1, \sin t + 2, \frac{2}{3}(t+1)^{3/2} + \frac{1}{3} \right\rangle \end{aligned}$$

(c) Is the motion of the particle 'smooth'? Explain.

The motion is smooth if  $\vec{s}'(t) = \vec{v}(t)$  is never  $\vec{0}$ .

$$\langle 3t, \cos(t), \sqrt{t+1} \rangle$$

will be 0 in the x component only when

$$t=0,$$

but then  $y, z$  are non zero.

So, yes, the motion is smooth

4. Give parametric equations for the curve of intersection of the cylinder  $y^2 + z^2 = 4$  and the plane  $x + y + z = 1$ . (Do more here than a 'brute force' setting of one variable to  $t$ ).

$$y^2 + z^2 = 4 \rightarrow \begin{matrix} \text{let} \\ y = 2\cos t \\ z = 2\sin t \end{matrix}$$

$$\text{Then } x = 1 - y - z = 1 - 2\cos t - 2\sin t$$

$$\text{So } \vec{r}(t) = \langle 1 - 2\cos t - 2\sin t, 2\cos t, 2\sin t \rangle$$

5. Extra Credit:

- (a) What is your birthday? (Month and Day only. No years. Please.)

Nov. 19 ☺

- (b) Of the 58 of us (myself included), what is the approximate probability that at least 2 of us have the same birthday?

i. 16 %

ii. 32 %

iii. 64 %

iv. 99 %