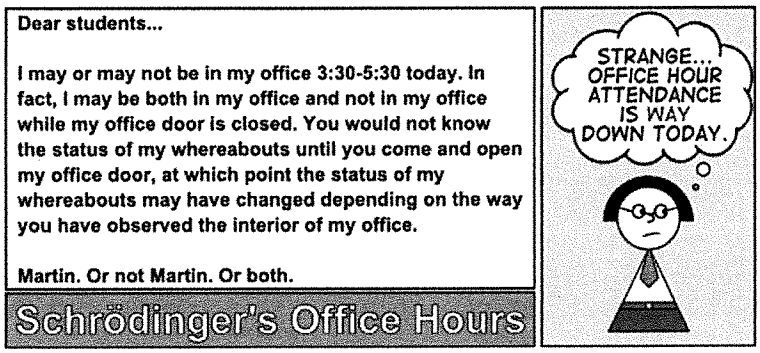
KEY

Math 225: Quiz the Fourth
September 29, 2017

You have the remainder of the period to complete this quiz. You may use a calculator for arithmetic only.



1. Identify each surface below, and provide the requested information.

   (a) $4x^2 + 9y^2 + 36z^2 = 144$ (give also max/min x-values)

   ellipsoid → $\frac{4x^2}{144} + \frac{9y^2}{144} + \frac{36z^2}{144} = 1$

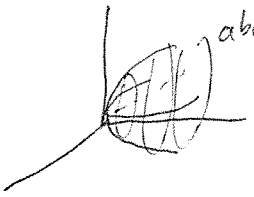
   $\frac{x^2}{36} + \frac{y^2}{16} + \frac{z^2}{4} = 1$ → $-6 \le x \le 6$

   (b) $x^2 - 9y^2 + 12z^2 = 1$ (give also the axis of symmetry)

   $x^2 + 12z^2 = 1 + 9y^2$ hyperboloid of one sheet
   & y-axis of symmetry

   (c) $x + y^2 + z^2 = 0$ (give a rough sketch of the graph)

   $x = -y^2 - z^2$ ← elliptical (circular) paraboloid

2. Consider the space curve $\mathbf{r}(t) = \langle t^2, 2t, t^2 - 1 \rangle$.

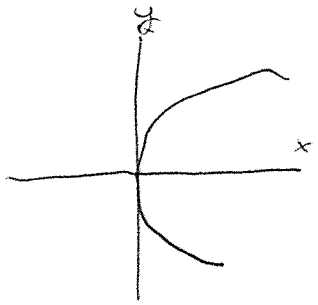
(a) What are the values that x, y, and z take on?

$x = t^2 \quad x \geq 0$

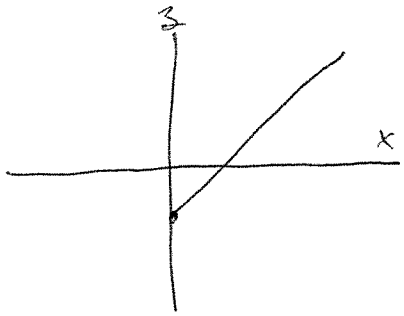
$y = 2t \quad y$ is free

$z = t^2 - 1 \quad z \geq -1$

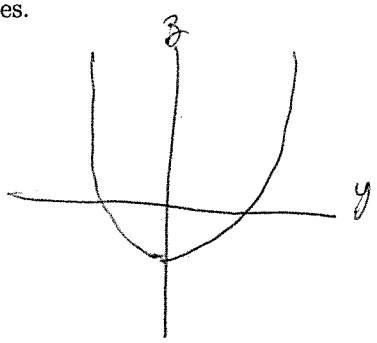
(b) Draw the views of the curve looking down the x, y and z-axes.



$x = \left(\frac{y}{2}\right)^2$



$z = x - 1$ for pos. x values



$z = \left(\frac{y}{2}\right)^2 - 1$

(c) Find the equation of the tangent line to $\mathbf{r}(t)$ when $t = 3$.

$\vec{r}(3) = \langle 9, 6, 8 \rangle$

$\vec{r}\,'(3) = \langle 2t, 2, 2t \rangle \Big|_{t=3} \rightarrow \langle 6, 2, 6 \rangle$

line: $\vec{l}(t) = \langle 9, 6, 8 \rangle + t\langle 6, 2, 6 \rangle$

3. Suppose that a particle in motion satisfies $\mathbf{v}(t) = \langle 3t, \cos(t), \sqrt{t+1} \rangle$ for $t \geq -1$ (where $\mathbf{v}(t)$ is the velocity vector).

(a) Find the acceleration $\mathbf{a}(t)$.

$\vec{a}(t) = \vec{v}'(t) = \langle 3, -\sin(t), \frac{1}{2\sqrt{t+1}} \rangle$

(b) Find the position, $\mathbf{s}(t)$, if $\mathbf{s}(0) = \langle 1, 2, 1 \rangle$

$s(t) = \int_0^t \vec{v}(r)\,dr + \langle 1, 2, 1 \rangle$

$= \left\langle \frac{3}{2}r^2, \sin r, \frac{2}{3}(t+1)^{3/2} \right\rangle \Big|_0^t + \langle 1, 2, 1 \rangle$

$= \langle \frac{3}{2}t^2, \sin t, \frac{2}{3}(t+1)^{3/2} \rangle - \langle 0, 0, \frac{2}{3} \rangle + \langle 1, 2, 1 \rangle$

$= \langle \frac{3}{2}t^2 + 1, \sin t + 2, \frac{2}{3}(t+1)^{3/2} + \frac{1}{3} \rangle$

(c) Is the motion of the particle 'smooth'? Explain.

The motion is smooth if $\vec{s}'(t) = \vec{v}(t)$ is never $\vec{0}$.

$\langle 3t, \cos(t), \sqrt{t+1} \rangle$

will be 0 in the x component only when

$t = 0$,

but then y, z are non zero.

So, yes, the motion is smooth

4. Give parametric equations for the curve of intersection of the cylinder $y^2 + z^2 = 4$ and the plane $x + y + z = 1$. (Do more here than a 'brute force' setting of one variable to t).

$y^2 + z^2 = 4 \rightarrow$ let $y = 2\cos t$, $z = 2\sin t$

Then $x = 1 - y - z = 1 - 2\cos t - 2\sin t$

So $\vec{r}(t) = \langle 1 - 2\cos t - 2\sin t, 2\cos t, 2\sin t \rangle$

5. Extra Credit:

(a) What is your birthday? (Month and Day only. No years. Please.) Nov. 19 :c

(b) Of the 58 of us (myself included), what is the approximate probability that at least 2 of us have the same birthday?

i. 16 % ii. 32 % iii. 64 % iv. 99 %