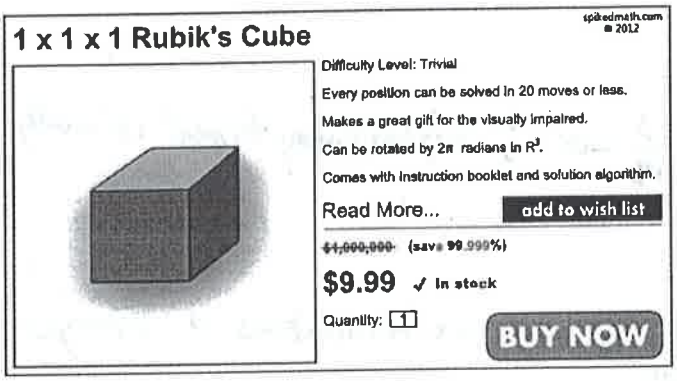
KEY

Math 225: Quiz the Fifth
October 20, 2017

You have the remainder of the period to complete this quiz. You may use a calculator for arithmetic only.



1. Let $f(x,y) = x^2 + y^3 + e^{xy}$

   (a) Find f_x and f_y.

$$f_x = 2x + ye^{xy}$$

$$f_y = 3y^2 + xe^{xy}$$

   (b) Find the equation of the tangent plane to $f(x,y)$ at the point (0,1).

$$f(0,1) = 0^2 + 1^3 + e^{0\cdot 1} = 2$$

$$f_x(0,1) = 2(0) + 1\cdot e^{0\cdot 1} = 1$$

$$f_y(0,1) = 3\cdot 1^2 + 0\cdot e^{0\cdot 1} = 3$$

Plane: $z = 2 + (x-0) + 3(y-1)$

3. The strength of an economy (S) might be loosely modeled on 3 factors: Productivity (p), Unemployment (u), and Relations with other countries (r). Each of these changes relative to time (t).

   (a) Intuitively, what should the sign be (and explain why!) on

   i. $\frac{\partial S}{\partial p}$ $\frac{\partial S}{\partial p} > 0$, more productivity = healthier economy

   ii. $\frac{\partial S}{\partial u}$ $\frac{\partial S}{\partial u} < 0$; higher unemployment is (usually) bad

   iii. $\frac{\partial S}{\partial r}$ $\frac{\partial S}{\partial r} > 0$; better relations → stronger economy

   (b) Find the derivative $\frac{dS}{dt}$ (pay attention to fonts!)

$$\frac{dS}{dt} = \frac{\partial S}{\partial p}\frac{dp}{dt} + \frac{\partial S}{\partial u}\frac{du}{dt} + \frac{\partial S}{\partial r}\cdot\frac{dr}{dt}$$

4. Find $\frac{\partial f}{\partial t}$ if $f(x,y) = x^4 + y^4 - \cos(xy)$, $x = s^2 - t^2$ and $y = 2st$. (You can leave your answer in terms of x, y, s and t).

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t}$$

$$= (4x^3 + y\sin(xy))\cdot(-2t) + (4y^3 + x\sin(xy))(2s)$$

2. A population of grouper satisfies the equation

$$G(s,m) = 5m - .1s^2 + 0.0003sm$$

where s is the population of sharks, and m is the population of minnows. Suppose that we have 50 sharks and 1000 minnows.

(a) How many grouper are present?

$G = 5(1000) - .1(50)^2 + 0.0003(50)(1000)$

$= 4765$

(b) What is the approximate change in the number of grouper if we add 10 sharks and remove 150 minnows from the population? (Use the differential here)

$s = 50 \quad ds = 10$

$m = 1000 \quad dm = (-150)$

$dG = \frac{\partial G}{\partial s}\,ds + \frac{\partial G}{\partial m}\,dm$

$= (-.2s + 0.0003m)\,ds + (5 + 0.0003s)\,dm$

$= (-10 + .3)(10) + (5 + 0.015)(-150)$

$= -849.25$ Grouper lost.

(c) (Bonus) Why (biologically) might the sign on the last term be positive? $(+0.0003sm)$

when sharks & minnows interact, sharks might eat minnows (instead of grouper!)

5. Let $xy + yz + xz = 14$.

(a) For which values (x, y, z) can we not define y as a function of x and z (Give your answer as an equation)

y will not be a function of x & z when $F_y = 0$

when $F(x,y,z) = xy + xz + yz$.

So $F_y = x + z = 0$

means y is not a fcn of x, z

(b) Find $\frac{\partial y}{\partial x}$ and $\frac{\partial y}{\partial z}$ at the point $(2, 4, 1)$

$\frac{\partial y}{\partial x} = \frac{-F_x}{F_y} = \frac{-(y+z)}{x+z}\Big|_{2,4,1} = \frac{-5}{3}$ $\quad \frac{\partial y}{\partial z} = \frac{-F_z}{F_y} = \frac{-(x+y)}{x+z} = \frac{-6}{3} = -2$

55

22 34 12
55 3 33
22 44 42
60 44 42
22 20 75
22 50 37
7 20 5
75 18 33
66 99 50
19 42 33
33 33 60
22 33 75
65 0 25
3 15 0
33 31 15
33 48
42 15
15 0
17 67
67 36

6. (Bonus) Choose a number between 0 and 100. I will give 1 point EC to the student who is closest to $\frac{2}{3}$ of the number chose. You may also earn up to 1 point EC for a justification of the number that you chose.

360
330
374
278
362
175

$\Sigma = 1879$ $\quad \bar{x} = 34.16$

$\frac{2}{3}\bar{x} = 22.77$