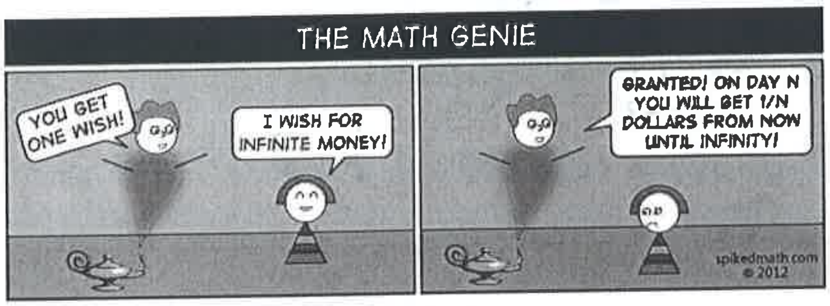
KEY

Math 225: Quiz the Sixth
October 27, 2017

You have the remainder of the period to complete this quiz. You may use a calculator for arithmetic only.



1. True or false. Give a brief justification in each case.

   (a) $D_{\mathbf{u}}f$ is a unit vector.

   FALSE: $D_u f = \vec{\nabla} f \cdot \vec{u}$, a scalar!

   (b) The gradient vector of a function f at a point (a,b) is orthogonal to the level curve of f at (a,b).

   True. In this case $D_{\vec{u}} f = 0 = \vec{\nabla} f \cdot \vec{u}$ so $\vec{\nabla} f \perp \vec{u}$.

   (c) The gradient vector at a local maximum of $f(x, y)$ is $\langle 0, 0\rangle$.

   True! To find a local max, set $f_x = f_y = 0$ so $\vec{\nabla} f = \langle 0, 0\rangle$.

   (d) A 'well-behaved' function has four second derivatives.

   True but ... $f_{xy} = f_{yx}$ so there are only 3 distinct derivatives

2. Find the directional derivative of the function $f(x, y) = x^2 + 2xy + y^3$ at the point (2,1) in the direction of the vector $\langle 3, -4 \rangle$.

$$\vec{\nabla} f = \langle 2x + 2y,\ 2y + 3y^2 \rangle \Big|_{(2,1)}$$

$$= \langle 6, 7 \rangle$$

$$\vec{u} = \frac{\langle 3, -4 \rangle}{|\langle 3, -4 \rangle|} = \left\langle \frac{3}{5}, \frac{-4}{5} \right\rangle$$

$$D_{\vec{u}} f = \langle 6, 7 \rangle \cdot \left\langle \frac{3}{5}, \frac{-4}{5} \right\rangle = \frac{18}{5} - \frac{28}{5} = \frac{-10}{5} = -2.$$

3. Let $f(x, y, z) = 5x^2 - 3xy + xyz$. Find the maximum rate of change at the point (5,4,3), and the direction that it occurs in.

$$\vec{\nabla} f = \langle 10x - 3y + yz,\ -3x + xz,\ xy \rangle$$

$$= \langle 50, 0, 20 \rangle$$

So $\max D_{\vec{u}} f = |\langle 50, 0, 20 \rangle| = \sqrt{2900}$

Direction: $\langle 50, 0, 20 \rangle$.

4. Suppose that $f(x, y)$ is a function such that $|\nabla f| = 4$. Find $D_{\mathbf{u}} f$ if the angle between $\mathbf{u}$ and ∇f is $\frac{\pi}{3}$ radians.

$$D_{\vec{u}} f = \vec{\nabla} f \cdot \vec{u} = |\vec{\nabla} f| |\vec{u}| \cos\theta$$

$$= 4 \cdot 1 \cdot \cos \pi/3$$

$$= 4 \cdot 1/2 = 2.$$

5. Suppose that a function $f(x, y)$ satisfies $f_x = x^2 + 2y$. Give at least three possibilities for f_y. (Your possibilities should differ by more than just a constant).

$f_x = x^2 + 2y \rightarrow f = \frac{x^3}{3} + 2xy + g(y) \rightarrow f_y = 0 + 2x + g'(y)$

So possible f_y's are

$f_y = 2x$

$f_y = 2x + 7y$

$f_y = 2x + e^y + \sqrt{y^{4.2} + 1}$ etc.,

6. Find and classify the critical points of $f(x, y) = 4 + x^3 + y^3 - 3xy$.

$f_x = 3x^2 - 3y = 0 \rightarrow y = x^2$

$f_y = 3y^2 - 3x = 0$

$3x^4 - 3x = 0$

$3(x)(x^3 - 1) = 0 \rightarrow x = 0, x = 1$ cp: $(0,0)$ $(1,1)$

$\rightarrow y = 0 \quad y = 1$

Classify:

$D = f_{xx} f_{yy} - (f_{xy})^2$

$= (6x)(6y) - (3)^2$

$(0,0)$: $D = -9 < 0 \rightarrow$ f has a

$(0,0)$ is a saddlepoint

$(1,1)$ $D = 36 - 9 > 0$

so f has a max or min

$f_{xx} > 0 \rightarrow$ f is concave up

$\rightarrow (1,1)$ is a local min.

7. Use the gradient vector to find the tangent plane to the surface $xy + yz + xz = 14$ at the point $(2, 1, 4)$. (You may, instead, for partial credit, solve for z and find the tangent plane this way).

$\vec{n} = \vec{\nabla} F = \langle y+z, x+z, x+y \rangle \big|_{(2,1,4)}$

$= \langle 5, 6, 3 \rangle$

point: $(2, 1, 4)$

plane: $5(x-2) + 6(y-1) + 3(z-4) = 0$

8. (Bonus) You can take either 0.5 EC for yourself, or 0 for yourself, and give 0.02 EC to everyone else in Calc 3. Your EC score will be the sum of what you take and what others give you.