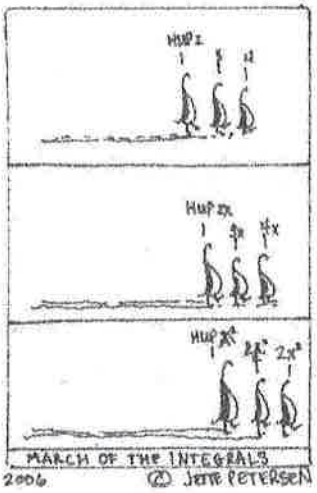
KEY

Math 225: Quiz the Seventh
November 3, 2017

You have the remainder of the period to complete this quiz. You may use a calculator for arithmetic only.



1. Find the maximum and minimum value of $f(x, y) = x+2y$ subject to the constraint $2x^2+y^2 = 18$

$x+2y$ s.t. $2x^2+y^2=18$

$1 = 4x\lambda$ $\quad x = 1/4\lambda$

$2 = 2y\lambda \rightarrow y = \frac{1}{\lambda}$

$2x^2+y^2=18$

$2\left(\frac{1}{4\lambda}\right)^2 + \left(\frac{1}{\lambda}\right)^2 = 18$

$\frac{2}{16\lambda^2} + \frac{16}{16\lambda^2} = 18$ $\quad \frac{18}{16\lambda^2} = 18$

$\lambda = \frac{\pm 1}{4} \rightarrow x = \pm 1$, $y = \pm 4$

Max: $f(1,4) = 9$

min: $f(-1,-4) = -9$.

2. The Cobb-Douglas production function is given by

$$f(l, c) = 100 l^{\frac{3}{4}} c^{\frac{1}{4}}$$

where l is the number of units of labor, and c is the number of units of capital used in production. Suppose that we have \$40,000 to spend between capital and labor, and that labor costs \$300 per unit and capital costs \$200 per unit. Find the amount to spend on labor and capital so as to maximize production.

Maximize

$f(l,c) = 100 l^{\frac{3}{4}} c^{\frac{1}{4}}$ s.t. $300l + 200c = 40{,}000$ (3)

$f_l = 75 l^{-\frac{1}{4}} c^{\frac{1}{4}} = 300\lambda$ (1)

$f_c = 25 l^{\frac{3}{4}} c^{-3/4} = 200\lambda$ (2)

from (1): $\lambda = \frac{1}{4} l^{-1/4} c^{1/4}$

so (2): $25 l^{\frac{3}{4}} c^{-3/4} = 200\left(\frac{1}{4} l^{-1/4} c^{1/4}\right)$

$\rightarrow 25l = 50c$

$l = 2c$

(3): $\rightarrow 300(2c) + 200(c) = 40{,}000$

$800c = 40{,}000$

$c = 50$

$l = 100$

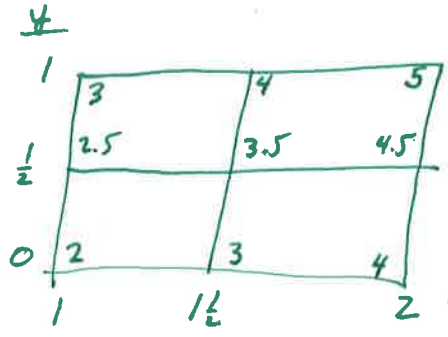
so, spend 30,000 on labor
10,000 on capital.

3. Give an over-estimate and an underestimate of

$$\int_1^2 \int_0^1 2x + y \, dy \, dx$$

using 4 rectangles.

R:



Box size: $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

under $= \frac{1}{4}(2 + 2.5 + 3 + 3.5) = \frac{11}{4}$

over $= \frac{1}{4}(3.5 + 4 + 4.5 + 5) = \frac{17}{4}$.

4. Find $\iint_R 4x + 2y + 3 \, dA$, where $R = [1, 2] \times [0, 3]$.

$$\int_1^2 \int_0^3 4x + 2y + 3 \, dy \, dx$$

$$= \int_1^2 4xy + y^2 + 3y \Big]_0^3 = \int_1^2 12x + 9 + 9 \, dx$$

$$= \int_1^2 12x + 18 \, dx = 6x^2 + 18x \Big]_1^2$$

$$24 + 36 - (6 + 18)$$

$$= \boxed{36}$$

5. Find $\iint_R xe^{xy}\, dA$, where $R = [0,2] \times [0,1]$

$$\int_0^2\int_0^1 x e^{xy}\, dy\, dx \rightarrow \int_0^2\int_0^1 e^u\, du\, dx$$

$u = xy$

$du = x\,dy$

$$= \int_0^2 e^{xy}\Big|_0^1\, dx$$

$$= \int_0^2 e^x - 1\, dx$$

$$= e^x - x\Big]_0^2$$

$$= (e^2 - 2) - 1 = \underline{e^2 - 3}$$

6. (Bonus) Pretend to toss a coin 5 times. Write your results as a string of Heads and Tails.

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