Math 225: Review Problems
Fall 2021

These are some sample problems from past exams. They are not meant to be exhaustive of what you will see on Wednesday's exam. We will go over these on Monday in class.

1. Draw each of the following parametric curves, each with attention to direction and allowable x and y values. All graphs are defined for all real t. (Hint: Eliminate the parameter)
   (a) $\mathbf{r}(t) = \langle t, 1-t \rangle$
   (b) $\mathbf{r}(t) = \langle -e^t, 1+e^t \rangle$
   (c) $\mathbf{r}(t) = \langle \sin^2(t), \cos^2(t) \rangle$
2. Consider the curve $x = 4t - t^2$, $y = 3t$.
   (a) Find the points (if they exist) where the curve has a vertical or horizontal tangent.
   (b) Find the area bound by this curve and the y-axis.
3. Consider the cardioid $r = 1 + \cos(\theta)$.
   (a) Find the equation of the tangent line to this cardioid at the point $(\frac{3}{4}, \frac{3\sqrt{3}}{4})$ (when $\theta = \frac{\pi}{3}$)
   (b) Find the area bound by this cardioid in the first quadrant.
4. Write the vector $\langle -1, 5, 3 \rangle$ as a sum of a vector *parallel* to $\langle 4, 2, 4 \rangle$ and a vector *perpendicular* to $\langle 4, 2, 4 \rangle$
5. Consider the points $A = (1, 4, 6)$, $B = (3, -2, 8)$ and $P = (x, y, z)$.
   (a) Write the following as a mathematical equation.
   $d(A, P) = d(B, P)$
   (b) Simplify your equation into the form of a plane and give (and comment on!) its normal vector.
   (c) Give the intersection point of this plane with each of the coordinate axes.
6. Consider the points $A = (3, 1, 0)$, $B = (4, -1, 2)$, and $C = (5, 3, 1)$.
   (a) Find the equation of line through A that is parallel to the line through B and C.
   (b) Show that $\triangle ABC$ is a right triangle.
   (c) Find the area of $\triangle ABC$ by
      i. Cross-Products
      ii. High School Geometry
7. Prove that if $\mathbf{a}$ and $\mathbf{b}$ are vectors of equal magnitude, then $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$ are orthogonal.
8. Suppose that $\mathbf{v}$ and $\mathbf{w}$ are vectors, with $|\mathbf{v}| = 4$ and $|\mathbf{w}| = 5$.
   (a) What are the maximum and minimum values of $\mathbf{v} \cdot \mathbf{w}$?

(b) What are the maximum and minimum lengths of $\mathbf{v} \times \mathbf{w}$?

9. (a) Find the equation of the plane through the point $(1, -1, -1)$ that is parallel to the plane $5x - y - z = 6$.

   (b) Find the line of intersection of the plane you found in part (a) with the plane $x+y-z = 1$.

10. Match the equation to the surface description. Warning: There is one outlier in each group!!

(a) $x^2 + 4y^2 + 16 = 16z$	I Hyperboloid of One Sheet
(b) $x^2 + 4y^2 + 16z^2 = 16$	II Hyperboloid of Two Sheets
(c) $x^2 - 4y^2 + 16 = 16$	III Ellipsoid
(d) $x^2 + 4y^2 + 16 = 16z^2$	IV Elliptical Paraboloid

11. Consider $\mathbf{r}(t) = \langle 2\cos(t), 2\sin(t), 3 - 4\cos(t) \rangle$

   (a) This curve is the intersection of which *cylinder* and which *plane*?

   (b) What are maximum and minimum allowable values for each of x, y, and z?

   (c) Find $\mathbf{T}(\frac{\pi}{2})$ for this curve.

   (d) Find the equation of the tangent line to this curve at $t = \dfrac{\pi}{2}$.