Exam Review

1.

a. $r(t) = \langle t, 1-t \rangle$ $\quad y = 1-x$

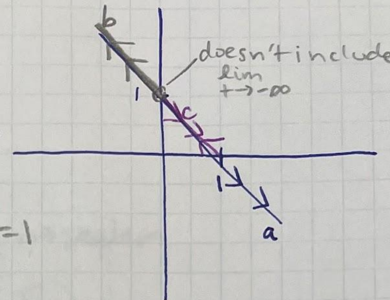
b. $r(t) = \langle -e^t, 1+e^t \rangle$ $\quad y = 1-x$

$x < 0 \quad y > 1$

c. $r(t) = \langle \sin^2(t), \cos^2(t) \rangle$ $\quad y = 1-x$

both between 0 + 1, oscillating

$\sin^2(t) + \cos^2(t) = 1$ so $x + y = 1$



2. $x = 4t - t^2 \quad y = 3t$

a. tangent $= \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

Horizontal when $\frac{dy}{dt} = 0$

Vertical when $\frac{dx}{dt} = 0$

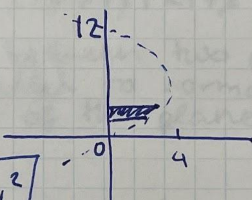
$\frac{dy}{dt} = 3$ no horizontal tangent

$\frac{dx}{dt} = 4 - 2t = 0 \quad t = 2$ @ point $(4, 6)$

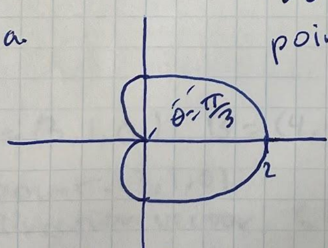
b. y-intercepts at $t=0$ and $t=4$

$A = \int x\,dy = \int_0^4 (4t - t^2)\,3\,dt$

$= 3\left[2t^2 - \frac{t^3}{4}\right]_0^4 = 3\left(32 - \frac{64}{3}\right) = \boxed{32\,u^2}$



3. a



point: $\left(\frac{3}{4}, \frac{3\sqrt{3}}{4}\right)$

slope $= \frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$

$= \frac{-\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{3}{2}\cdot\frac{1}{2}}{-\frac{\sqrt{3}}{2}\cdot\frac{1}{2} - \frac{3}{2}\cdot\frac{\sqrt{3}}{2}} = \frac{-\frac{3}{4} + \frac{3}{4}}{\sim} = 0$

$r = 1 + \cos\theta = 3/2$

$\frac{dr}{d\theta} = -\sin\theta = -\frac{\sqrt{3}}{2}$

$\sin\theta = \sqrt{3}/2$

$\cos\theta = \frac{1}{2}$

$\therefore$ Horizontal tangent line $\boxed{y = \frac{3\sqrt{3}}{4}}$

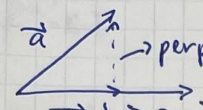
b. A in 1st quadrant

$A = \frac{1}{2}\int_0^{\pi/2} (f(\theta))^2\,d\theta = \frac{1}{2}\int_0^{\pi/2} (1+\cos\theta)^2\,d\theta = \frac{1}{2}\int_0^{\pi/2} 1 + 2\cos\theta + \cos^2\theta\,d\theta$

$= \frac{1}{2}\int_0^{\pi/2} 1 + 2\cos\theta + \frac{1}{2} + \frac{\cos 2\theta}{2}\,d\theta = \frac{1}{2}\left[\theta + 2\sin\theta + \frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_0^{\pi/2} = \frac{1}{2}\left(\frac{\pi}{2} + 2 + \frac{\pi}{4} + 0\right)$

$= \frac{3\pi}{8} + 1$

4. Write vector $\overset{\vec{a}}{\langle -1,5,3\rangle}$ as sum of vector $\parallel$ to $\langle 4,2,4\rangle$ and vector $\perp$ to $\langle 4,2,4\rangle$



$\to$ perpendicular vector

$\vec{b}$ $\hookrightarrow$ $\text{proj}_{\vec{b}}\vec{a}$ gives $\parallel$ vector

$$\text{proj}_{\vec{b}}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\cdot\vec{b} = \frac{-4+10+12\vec{b}}{(\sqrt{16+4+16})^2} = \frac{18\vec{b}}{36} = \cancel{\tfrac{1}{4}}\ \tfrac{1}{2}\ \text{scalar}$$

$\frac{1}{2}\vec{b}$

$\langle -1,5,3\rangle = \langle 2,1,2\rangle + \langle -3,4,1\rangle$

$\langle 2,1,2\rangle$ ↓ parallel; $\langle -3,4,1\rangle$ ↓ perpendicular

5. $A=(1,4,6)$ $B=(3,-2,8)$ $P=(x,y,z)$

dist. A–P = dist. B–P

a. $\sqrt{(x-1)^2+(y-4)^2+(z-6)^2} = \sqrt{(x-3)^2+(y+2)^2+(z-8)^2}$

$\cancel{x^2}-2x+1+\cancel{y^2}-8y+16+\cancel{z^2}-12z+36 = \cancel{x^2}-6x+9+\cancel{y^2}+4y+4+\cancel{z^2}-16z+64$

$4x-12y+4z=24$

b. or $x-3y+z=6$

Vector between two points is parallel to normal vector of the plane. ↓ $\langle 2,-6,2\rangle$

c. Intersections: $(6,0,0)$, $(0,-2,0)$, $(0,0,6)$ } points that satisfy the equation for the plane

6. $A=(3,1,0)$ $B=(4,-1,2)$ $C=(5,3,1)$

a. point: $(3,1,0)$

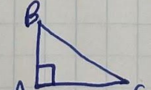
direction vector: $\langle 1,4,-1\rangle \to$ same as between B+C

line: $x=3+t$

$y=1+4t$

$z=-t$

b. $\overrightarrow{AB}=\langle 1,-2,2\rangle$ $\quad\overrightarrow{AB}\cdot\overrightarrow{AC}=0$ so it is a right triangle



$\overrightarrow{AC}=\langle 1,4,-1\rangle$

$\cancel{\overrightarrow{AB}}\ \overrightarrow{BC}=\langle 1,4,-1\rangle$

c. $\overrightarrow{AB}\times\overrightarrow{AC}$

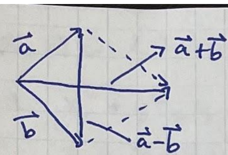
$$\begin{array}{r} \langle 1,-2,2\rangle \\ \times\ \langle 2,2,1\rangle \\ \hline \langle -6,3,6\rangle \end{array}$$

$A=|\overrightarrow{AB}\times\overrightarrow{AC}|\cdot\frac{1}{2}=\frac{9}{2}$

$A=\frac{1}{2}Bh \to |\overrightarrow{AB}|$, $\to |\overrightarrow{AC}|$

$A=\frac{9}{2}$

7.



$|\vec{a}| = |\vec{b}|$

$(\vec{a}+\vec{b}) \cdot (\vec{a}-\vec{b}) = \vec{a}\cdot\vec{a} - \cancel{\vec{a}\cdot b} + \cancel{\vec{a}\cdot b} - \vec{b}\cdot\vec{b}$

$= |\vec{a}|^2 - |b|^2 = 0$ b/c $|\vec{a}| = |\vec{b}|$

8. $|\vec{v}| = 4$ $|\vec{w}|$ 5

$|\vec{v}||\vec{w}|\cos\theta$

a. $-20 \le \vec{v}\cdot\vec{w} \le 20$ → $\vec{v}$ $\vec{w}$ parallel

V + w are antiparallel

b. V × W

$0 \le |\vec{v}\times\vec{w}| \le 20$

9. a. point $(1,-1,-1)$ parallel to $5x-y-z=6$ (plane)

point: $(1,-1,-1)$ $\vec{n}$ same as ↑ so $\langle 5,-1,-1\rangle$

plane = $\boxed{5(x-1)-(y+1)-(z+1)=0}$

or $\boxed{5x-y-z=7}$

b. Find intersection of $5x-y-z=7$ and $x+y-z=1$

dir. $\vec{v}$ is ~~plane × plane~~ cross of $\vec{n}_1$ and $\vec{n}_2$

$$\begin{array}{r} \langle 5,-1,-1\rangle \\ \times\ \langle 1,1,-1\rangle \\ \hline \langle 2,4,6\rangle \end{array}$$

point: $x=0$

$-y-z=7$

$y-z=1$

$-2z=8$

$z=-4$

$x+y-z=1$

$y+4=1$

$y=-3$

$(0,-3,-4)$

$\boxed{\vec{l}(t) = \langle 2t, -3+4t, -4+6t\rangle}$

related: distance between planes $(1,-1,-1)$ to $5x-y-z=6$ $d = \dfrac{|5(1)-1(-1)-1(-1)-6|}{\sqrt{5^2+(-1)^2+(-1)^2}} = \dfrac{1}{\sqrt{27}}$

10. a. $x^2+4y^2+16=16z$ → IV elliptical paraboloid up 16 units, shifted toward y

b. $x^2+4y^2+16z^2=16$ → III ellipsoid

c. $x^2-4y^2+16=16$ → $x=\pm 2y$ No match (degenerate hyperboloid)

d. $x^2+4y^2+16=16z^2$ → II Hyperboloid of 2 sheets

11.

11. $\vec{r}(t) = \langle 2\cos t,\ 2\sin t,\ 3-4\cos t \rangle$

a. $x^2+y^2=4 \longrightarrow$ cylinder

$z = 3-2x$ or $2x+z=3 \rightarrow$ plane

b. $-2 \leq x \leq 2$

$-2 \leq y \leq 2$

$-1 \leq z \leq 7 \longrightarrow$ centered around 3, goes up+down 4 units

c. $\vec{T}(\frac{\pi}{2}) = \frac{r'(t)}{|r'(t)|}$ $\vec{r}'(t) = \langle -2\sin t, 2\cos t, 4\sin t \rangle$ @ $\pi/2$

tangent unit vector

$= \langle -2, 0, 4 \rangle$

$$\boxed{\vec{T}(t) = \langle \frac{-2}{\sqrt{20}}, 0, \frac{4}{\sqrt{20}} \rangle}$$

d. $\vec{r}(\frac{\pi}{2}) = \langle 0, 2, 3 \rangle$

line: $\frac{x}{-2} = \frac{z-3}{4};\ y=2$

point