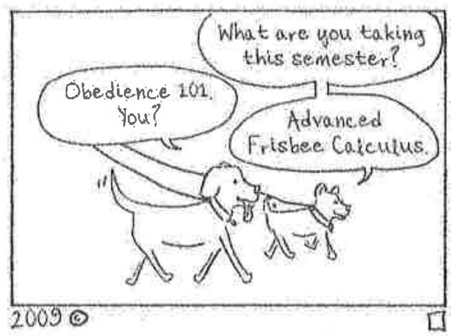
KEY

Math 225: Quiz the First
September 10, 2021

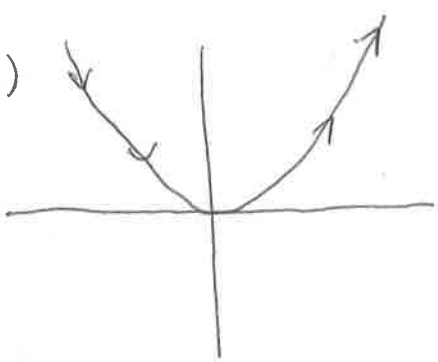
This quiz is closed book and closed notes. You may use a calculator for arithmetic only on this quiz (i.e., no graphing and no calculus). Please justify all of your answers. You have the remainder of the period.



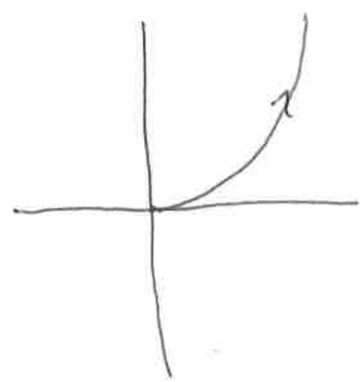
1. What are the similarities and differences between each of the following parametric curves?
   (a) $x = t,\ y = t^2, t \in R$
   (b) $x = \sqrt{t},\ y = t, t > 0$
   (c) $x = \cos(t),\ y = \cos^2(t), t \in R$

Each pair satisfies $y = x^2$, but each has a diffent domain, range

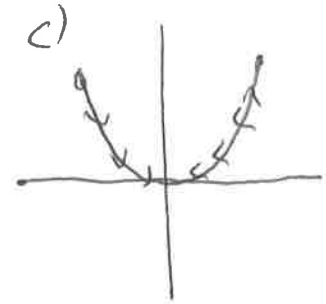
a)



b)



c)



full parabola
$y = x^2$

$x = \sqrt{t} \Rightarrow x > 0$

$-1 \leq x \leq 1$
$0 \leq y \leq 1$

2. Consider the curve $x(t) = 2t + 1, y(t) = 6t - t^2$

(a) Find the *points* where the curve crosses the y-axis (if any)

$x = 0 \rightarrow x = 2t + 1 = 0 \; , \; t = -1/2$ $\qquad$ $(0, -3\tfrac{1}{4})$

$y(-1/2) = 6(-\tfrac{1}{2}) - (\tfrac{1}{2})^2 = -3\tfrac{1}{4}$

(b) Find the *points* where the curve has a horizontal or vertical tangent (if any).

horiz tangent:

$\frac{dy}{dt} = 0, \; 6 - 2t = 0, \; t = 3$ $\qquad$ $x = 2(3)+1 = 7$, $\; y = 6(3) - 3^2 = 9$

$(7, 9)$

vert tangent:

$\frac{dx}{dt} = 0, \; 2 = 0$ ↯

no vert tangent.

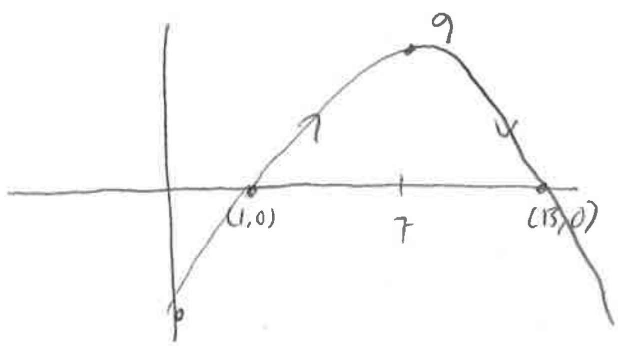
(c) Find the equation of the tangent line to the curve when $t = 1$. (Point-slope form is fine)

$t = 1$ $\qquad$ point: $((2(1)+1), 6(1) - 1^2) = (3, 5)$

slope $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{6-2t}{2}\Big|_{t=1} \rightarrow \frac{4}{2} = 2$

$\boxed{\text{line: } y - 5 = 2(x-3)}$

(d) Draw a rough sketch of the curve, with attention to direction.



(e) Find the area bound by the curve and the y-axis.

$$\int y\,dx = \int_0^6 (6t - t^2) \cdot 2\,dt$$

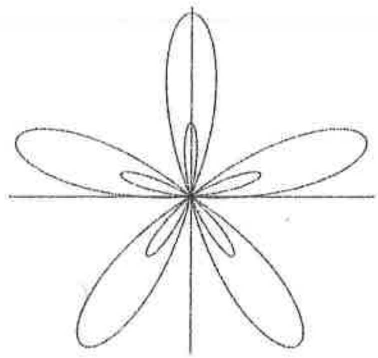
$$= \int_0^6 12t - 2t^2\,dt = 6t^2 - \frac{2}{3}t^3 \Big]_0^6$$

$$= 216 - \frac{2}{3}(216)$$

$$= \frac{216}{3} = 72 \text{ units}^2$$

3. Below is the plot of the polar curve

$$r = 3 + 7sin(5\theta)$$



(a) How 'long' are the large and small (outer and inner) leaves, respectively?

max r value $\Rightarrow 3+7=10$ ← large leaf

min r-value $\rightarrow 3-7=-4$ ← small leaf.

(b) At what values θ between 0 and π (inclusive) does this graph cross the origin?

$3+7\sin(5\theta)=0$

$5\theta = \arcsin\left(\frac{-3}{7}\right)$

$\theta = \frac{\arcsin\left(\frac{-3}{7}\right)}{5} + \frac{\pi}{5}, \quad \frac{\arcsin\left(\frac{-3}{7}\right)}{5} + 2\pi/5$

$+3\pi/5$

$+4\pi/5$

$+5\pi/5.$

$5\theta = -.4429$

$\theta = -.08$

(c) How would this graph compare to the graph of

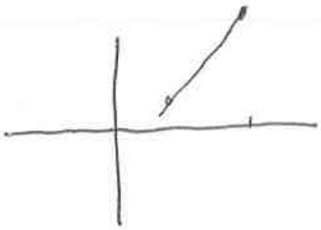
$$r = 3 + 7sin(7\theta)$$

The graph would be the same, it would have 7 leaves rather than 5.

Also, the leaf on the y-axis would point downward.

4. Give parametric equations (and appropriate t values!) for the following graphs. Note: More than one answer is possible, but please give only one.

   (a) A line between the points (1,2) and (3, 5).

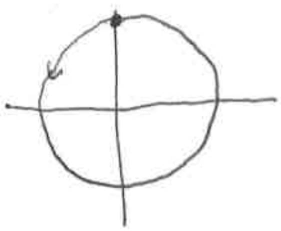


$x = x_0 + \Delta x \cdot t = 1 + 2t$

$y = y_0 + \Delta y \cdot t = 2 + 3t$

$-\infty \le t \le \infty$

   (b) A circle of radius 2, centered at the origin, traced counterclockwise once, starting at the point (0,2).

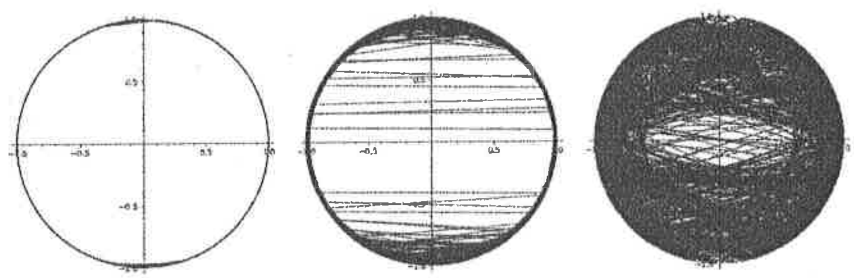


$x = 2\cos t$
$y = 2\sin t$
$\frac{\pi}{2} \le t \le \frac{5\pi}{2}$

or

$x = -2\sin t$
$y = 2\cos t$
$0 \le t \le 2\pi$.

5. (Bonus) See the graphs below. They represent the graph of the polar equations $x = \cos(t)$ and $y = \sin(t)$. The three graphs have increasing bounds on t. Why might we get different pictures for the graphs?



(The ranges on t are $(0, 20\pi)$, $(0, 200\pi)$, and $(0, 2000\pi)$.

When a computer draws a graph, it plots points and connects those points. If the range is large, the points may be further apart, so more "imperfections" may show in the drawings.