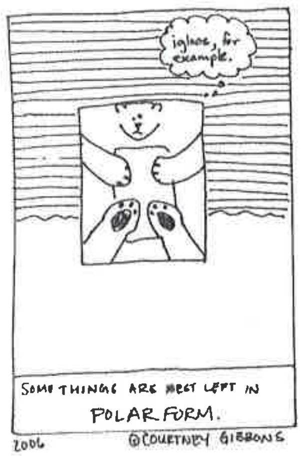
KEY

Math 125: Quiz the Second
September 17, 2021

You have the remainder of the period to complete this quiz. Please justify your answers where appropriate and READ ALL DIRECTIONS CAREFULLY. You may use a calculator for computation only.



1. Suppose $r = f(\theta)$. Find the equation for the slope of the graph of f at a point (x, y)

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

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2. Find the slope of $f(\theta) = cos(2\theta)$ when $\theta = \frac{\pi}{4}$

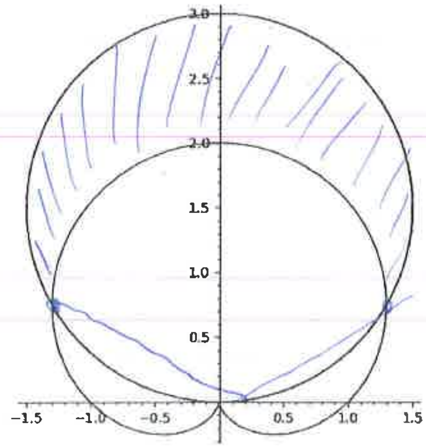
if $\theta = \frac{\pi}{4}$, $\cos 2\theta = \cos\frac{\pi}{2} = 0$

$\frac{dr}{d\theta} = -2\sin 2\theta = -2\sin\left(\frac{\pi}{2}\right) = -2$

$\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$$\frac{dy}{dx} = \frac{(-2)\frac{\sqrt{2}}{2} + 0\cdot\frac{\sqrt{2}}{2}}{(-2)\frac{\sqrt{2}}{2} - 0\,\frac{\sqrt{2}}{2}} = 1$$

3. The graphs of $r = 3\sin(\theta)$ and $r = 1 + \sin(\theta)$ are given below.



Set up, but don't evaluate, an integral or integrals to determine the area inside ~~$r = \sin(3\theta)$~~ but outside $r = 1 + \sin(\theta)$. Pay attention to the bounds on these integrals!

Bounds: $3\sin\theta = 1 + \sin\theta$

$\rightarrow 2\sin\theta = 1$

$\sin\theta = 1/2 \rightarrow \theta = \frac{\pi}{6},\ \theta = 5\pi/6$

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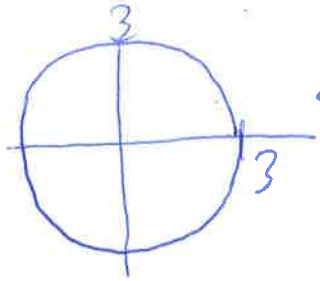
$$\frac{1}{2}\int_{\pi/6}^{5\pi/6} (f_2(\theta))^2 - (f_1(\theta))^2\, d\theta = \frac{1}{2}\int_{\pi/6}^{5\pi/6} (3\sin\theta)^2 - (1+\sin\theta)^2\, d\theta$$

4. Sketch and describe the graph of $x^2 + y^2 = 9$ in

(a) $\mathbb{R}^2$



← circle of radius 3 about the origin.

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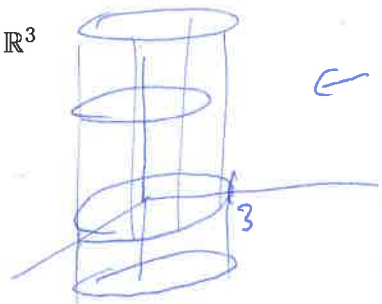
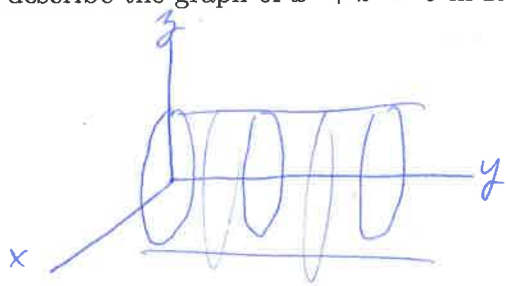
(b) $\mathbb{R}^3$



← circular cylinder parallel to z-axis.

5. Sketch and describe the graph of $x^2 + z^2 = 9$ in $\mathbb{R}^3$



← circular cylinder parallel to y-axis.

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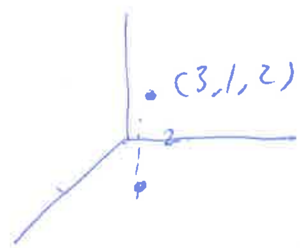
6. Find the distance from the point $(3, 1, 2)$ to

(a) The xy-plane

$d((3,1,2), (3,1,0)) = 2$



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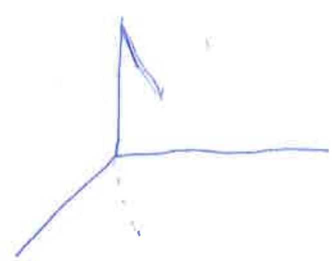
(b) The z-axis

$d((3,1,2),(0,0,2))$

$\sqrt{3^2+1^2+0^2} = \sqrt{10}$



7. Let $\mathbf{v} = \langle 1, -2, 2 \rangle$ and $\mathbf{w} = \langle 3, 4, -2 \rangle$

(a) Find the magnitudes of $\mathbf{v}$, $\mathbf{w}$ and $\mathbf{v} + \mathbf{w}$

$|\vec{v}| = \sqrt{1^2+2^2+2^2} = \sqrt{9} = 3$

$|\vec{w}| = \sqrt{3^2+4^2+(-2)^2} = \sqrt{29}$

$|\vec{v}+\vec{w}| = |\langle 3+1, -2+4, 2+-2 \rangle|$

$= |\langle 4, 2, 0 \rangle| = \sqrt{4^2+2^2+0} = \sqrt{20}$

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(b) Find a *unit vector* in the direction opposite (anti-parallel) to $\mathbf{v}$.

$|\vec{v}| = 3$ $\quad \vec{u}_{(same)} = \langle \frac{1}{3}, \frac{-2}{3}, \frac{2}{3} \rangle$

$\vec{u}_{opp} = \langle \frac{-1}{3}, \frac{2}{3}, \frac{-2}{3} \rangle$

8. (Bonus) Suppose that a standard clock has 11 vectors on it, each with its tail on the 6, and each with its head on a different number. What is the sum of these vectors?