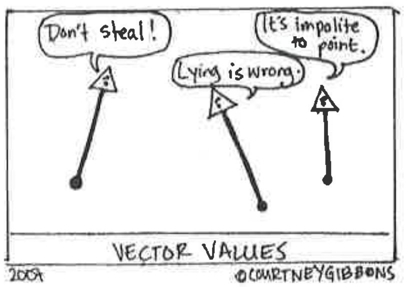
Key

Math 125: Quiz the Third

September 24, 2021

You have the remainder of the period to complete this quiz. Please justify your answers where appropriate and READ ALL DIRECTIONS CAREFULLY. You may use a calculator for computation only.



1. Suppose $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are vectors in $\mathbb{R}^3$. For each quantity below, determine whether it is a vector, a scalar, or nonsense.

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(a) $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$

vector • vector = scalar

(b) $(\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c}$

Scalar × vector = nonsense

(c) $\mathbf{a} \times (\mathbf{b} + \mathbf{c})$

vector × (vector) = vector

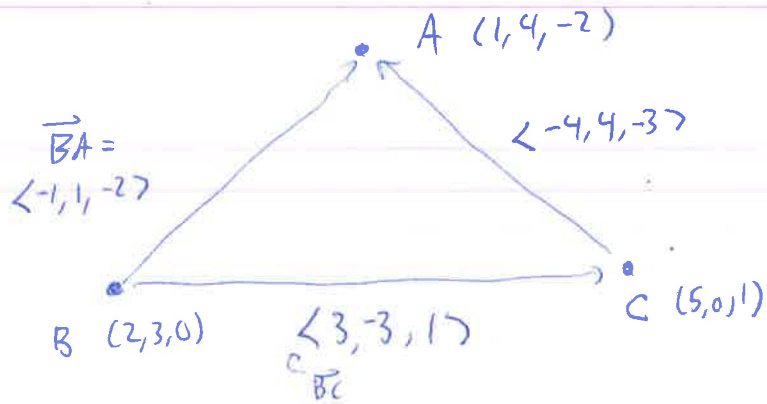
(d) $\mathbf{b}^2$

nonsense ... we don't square vectors.

2. Let ABC be a triangle with vertices

$$A = (1, 4, -2), \quad B = (2, 3, 0), \quad C = (5, 0, 1)$$

(a) Find the lengths of the sides of the triangle.



$AB = \sqrt{6}$
$BC = \sqrt{19}$
$AC = \sqrt{41}$

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(b) Is the angle at B acute, right, or obtuse?

$\vec{BA} \cdot \vec{BC} = \langle -1, 1, -2 \rangle \cdot \langle 3, -3, 1 \rangle = -3 - 3 - 2 = -8$

$\vec{BA} \cdot \vec{BC} < 0 \Rightarrow$ Angle is obtuse

3

(c) Find the area of triangle ABC.

$\text{Area} = \frac{1}{2} |\vec{BA} \times \vec{BC}| = \frac{1}{2} \left| \begin{matrix} \langle -1, 1, -2 \rangle \\ \times \langle 3, -3, 1 \rangle \end{matrix} \right|$

3 $\qquad = \frac{1}{2} |\langle -5, -5, 0 \rangle| = \frac{1}{2} \sqrt{25 + 25} = \frac{5\sqrt{2}}{2}$

*Please leave your answers as radicals.

3. Let $\mathbf{a} = \langle 3, 2, 1 \rangle$ and $\mathbf{b} = \langle -2, 1, 2 \rangle$. Find $\mathbf{proj_b a}$ and $\text{comp}_\mathbf{b} \mathbf{a}$.

$$\text{proj}_{\vec{b}} \vec{a} = \frac{(\vec{a} \cdot \vec{b})}{|\vec{b}|^2} \vec{b} = \frac{(-6+2+2)}{\left(\sqrt{-2^2+1^2+2^2}\right)^2} \vec{b} = \frac{-2}{9} \vec{b}$$
$$= \left\langle \frac{4}{9}, \frac{-2}{9}, \frac{-4}{9} \right\rangle$$

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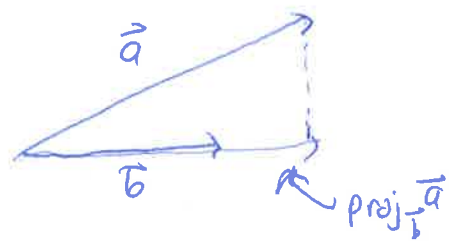
$$\text{Comp}_{\vec{b}}(\vec{a}) = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{-2}{3}$$

4. (a) Can it ever be the case that $\mathbf{proj_b\ a}$ is longer than $\mathbf{b}$? Give a geometric (drawing) or algebraic argument.

Yes



$\vec{a}$ can extend past $\vec{b}$,

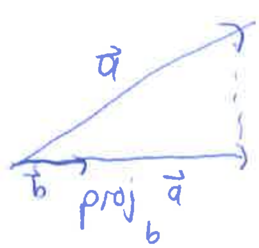
$\frac{\vec{a} \cdot \vec{b}}{\vec{b}}$ can be > 1, etc.

3

(b) Can it ever be the case that $\mathbf{proj_b\ a}$ is longer than $\mathbf{a}$? Give a geometric (drawing) or algebraic argument.

No

3



Since $\vec{a}$ is the hypotenuse of a right triangle, it is never longer than a leg.

5. Find a unit vector perpendicular to $\langle 3, 2, 1\rangle$ and $\langle -1, -4, 1\rangle$.

$$\begin{array}{r} \langle 3, 2, 1 \rangle \\ \times \ \langle -1, -4, 1 \rangle \\ \hline \langle 2 - {-4}, -1-3, -12 - {-2} \rangle \end{array}$$

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$$= \langle 6, -4, -10 \rangle \xrightarrow[\sqrt{36+16+100}]{\text{unit}} = \left\langle \frac{6}{\sqrt{152}}, \frac{-4}{\sqrt{152}}, \frac{-10}{\sqrt{152}} \right\rangle$$

6. (Bonus).Prove the parallelogram law:

$$|\mathbf{a} + \mathbf{b}|^2 + |\mathbf{a} - \mathbf{b}|^2 = 2|\mathbf{a}|^2 + 2|\mathbf{b}|^2$$

$$|a+b|^2 = (\vec{a}+\vec{b}) \cdot (\vec{a}+\vec{b}) = \vec{a}\cdot\vec{a} + \cancel{2\vec{a}\cdot\vec{b}} + \vec{b}\cdot\vec{b}$$
$$|\vec{a}-\vec{b}|^2 = (\vec{a}-\vec{b}) \cdot (a-\vec{b}) = \vec{a}\cdot\vec{a} - \cancel{2\vec{a}\cdot\vec{b}} + \vec{b}\cdot\vec{b}$$

add

$$2\vec{a}\cdot\vec{a} + 2\vec{b}\cdot\vec{b} = 2|\vec{a}|^2 + 2|\vec{b}|^2.$$

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