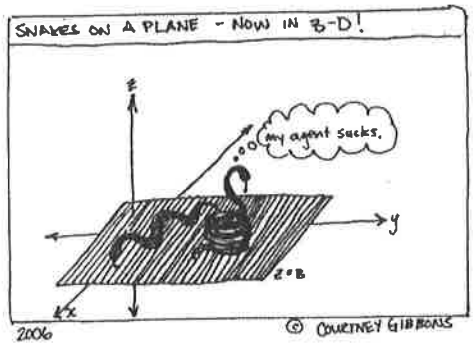
Key

Math 125: Quiz the Fourth
October 1, 2021

You have the remainder of the period to complete this quiz. Please justify your answers where appropriate and READ ALL DIRECTIONS CAREFULLY. You may use a calculator for computation only.



(2) 1. Find the equation(s) of the line through the point (3,1,2) and in the direction of $\langle -2, 1, 4\rangle$. Give *at least two* forms for this equation.

$\vec{r}(t) = \langle 3,1,2\rangle + t\langle -2,1,4\rangle$

$x = 3-2t$
$y = 1+t$
$z = 2+4t$

$\frac{x-3}{-2} = \frac{y-1}{1} = \frac{z-2}{4}$

(2) 2. Find the equation of the plane through the point (3,1,2) and normal to $\langle -2, 1, 4\rangle$

$-2(x-3) + 1(y-1) + 4(z-2) = 0$

3. Find the equation of the plane containing the points

$$A = (1, 4, -2), \quad B = (2, 3, 0), \quad C = (5, 0, 1)$$

$\vec{AB} = \langle 1, -1, 2 \rangle$

$\vec{AC} = \langle 4, -4, 3 \rangle$

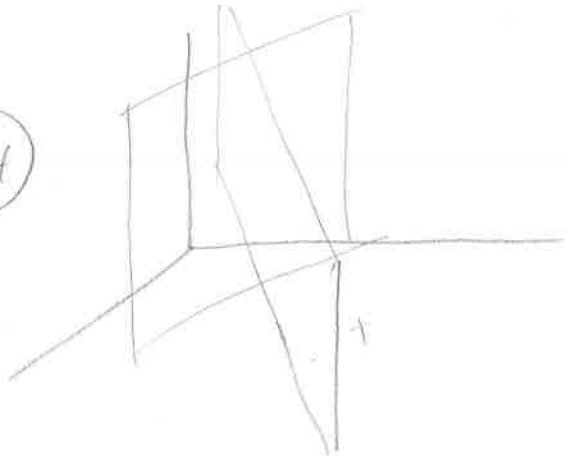
$\vec{AB} \times \vec{AC} = \langle 5, 5, 0 \rangle$

$5(x-1) + 5(y-4) = 0$

$5x + 5y = 25$, $x + y = 5$

(4)

4. Find two planes perpendicular to the xy-plane that are *not* perpendicular to each other.



(4)

xy plane → $z = 0$, $\vec{n} = \langle 0, 0, 1 \rangle$

Examples: $x + y = 1$

$x + 2y = 5$

any two planes not orthogonal with z-coefficient 0 will work

5. The following lines intersect

$$x = 3 + t, \quad y = -1 + t \quad z = 3 - 2t$$
$$x = 1 + 4s \quad y = -1 + 2s \quad z = 2 - 3s$$

(a) Find the point of intersection.

$3+t = 1+4s$
$-1+t = -1+2s \rightarrow t = 2s$
$3-2t = 2-3s$

$3+2s = 1+4s \rightarrow 2 = 2s$
$s = 1$
$t = 2$

$3-2(2) = 2-3(1) = -1$ ✓

Point:
$3+2 = 5$
$-1+2 = 1$
$3-2(2) = -1$

$(5, 1, -1)$

(3)

(b) Find the *acute* angle of intersection (you may leave your answer as an arccosine).

$\vec{v}_1 = \langle 1, 1, -2 \rangle$
$\vec{v}_2 = \langle 4, 2, -3 \rangle$

$\cos\theta = \dfrac{\vec{v}_1 \cdot \vec{v}_2}{|\vec{v}_1||\vec{v}_2|} = \dfrac{4+2+6}{\sqrt{6}\sqrt{29}}$

$\theta = \arccos\left(\dfrac{12}{\sqrt{6}\sqrt{29}}\right)$

(3)

6. Identify each surface below, and, for each, determine the allowable x values.

(a) $18x^2 + 72y^2 + 2z^2 = 288$

$\frac{x^2}{16} + \frac{y^2}{4} + \frac{z^2}{144} = 1$ Ellipsoid

$-4 \le x \le 4$

3

(b) $18x^2 - 72y^2 - 2z^2 = 288$ Hyperboloid of 2 sheets

$\frac{x^2}{16} = 1 + \frac{y^2}{4} + \frac{z^2}{144}$

$x \le -4$ or $x \ge 4$

2

(c) $-18x^2 + 72y^2 + 2z^2 = 288$ Hyperboloid of one sheet

$\frac{y^2}{4} + \frac{z^2}{144} = 1 + \frac{x^2}{16}$ → any x value allowed

2

7. (Bonus) Which mathematical terms come from the following roots?

(a) Latin from Greek for 'to throw alongside' Parabola

(b) Latin for 'to touch' tangent

(c) Arabic 'a'awareya', which refers to goods damaged at sea. average