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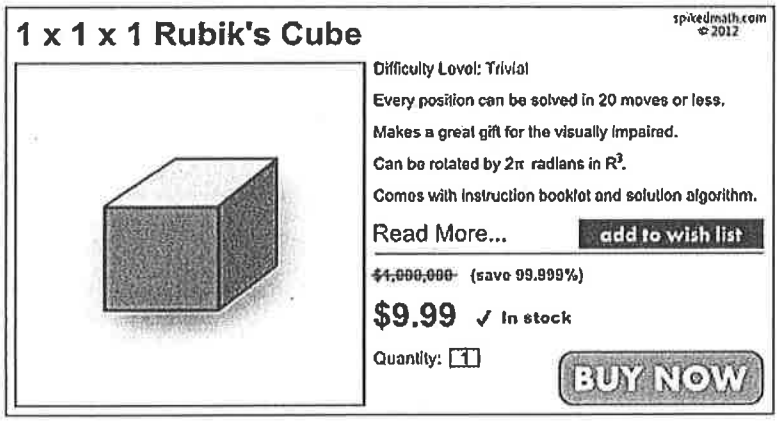
KEY

Math ~~22~~: Quiz the Fifth

October 22, 2021

You have the remainder of the period to complete this quiz. Please justify your answers where appropriate and READ ALL DIRECTIONS CAREFULLY. You may use a calculator for computation only.



1. An object is traveling with velocity vector

$$\mathbf{v} = \langle 3, 3t^2, -5t \rangle$$

Find the position and acceleration functions given that the initial position is $\langle 0, 0, 20 \rangle$

$\vec{a} = \vec{v}' = \langle 0, 6t, -5 \rangle$

$\vec{r}(t) = \int_0^t \vec{v}(t) + \vec{r}_0 = \langle 3t, t^3, -\frac{5}{2}t^2 \rangle + \langle 0, 0, 20 \rangle$

2. Suppose that a rocket is traveling with position function $\mathbf{r}(t) = \langle 6t, t^3, t^2 + 4t \rangle$ Find a_T, a_N and κ when $t = 1$.

$\vec{r} = \langle 6t, t^3, t^2+4t \rangle$

$\vec{r}' = \langle 6, 3t^2, 2t+4 \rangle|_{t=1} \rightarrow \langle 6, 3, 6 \rangle$

$\vec{r}'' = \langle 0, 6t, 2 \rangle|_{t=1} \rightarrow \langle 0, 6, 2 \rangle$

$$a_T = \frac{\vec{r}' \cdot \vec{r}''}{|\vec{r}'|} = \frac{30}{9}$$

$$a_N = \frac{|\vec{r}' \times \vec{r}''|}{|\vec{r}'|} = \frac{|\langle 30, -12, 36 \rangle|}{9} = \frac{6|\langle 5, -2, 6 \rangle|}{9}$$

$$= 6\frac{\sqrt{65}}{9}$$

$$\kappa = \frac{6\sqrt{65}}{729}$$

3. Let

$$f(x,y) = \frac{1}{1+x^2+y^2}$$

(a) What is the domain of f?

All real x, y.

(b) What shape are the level curves of f?

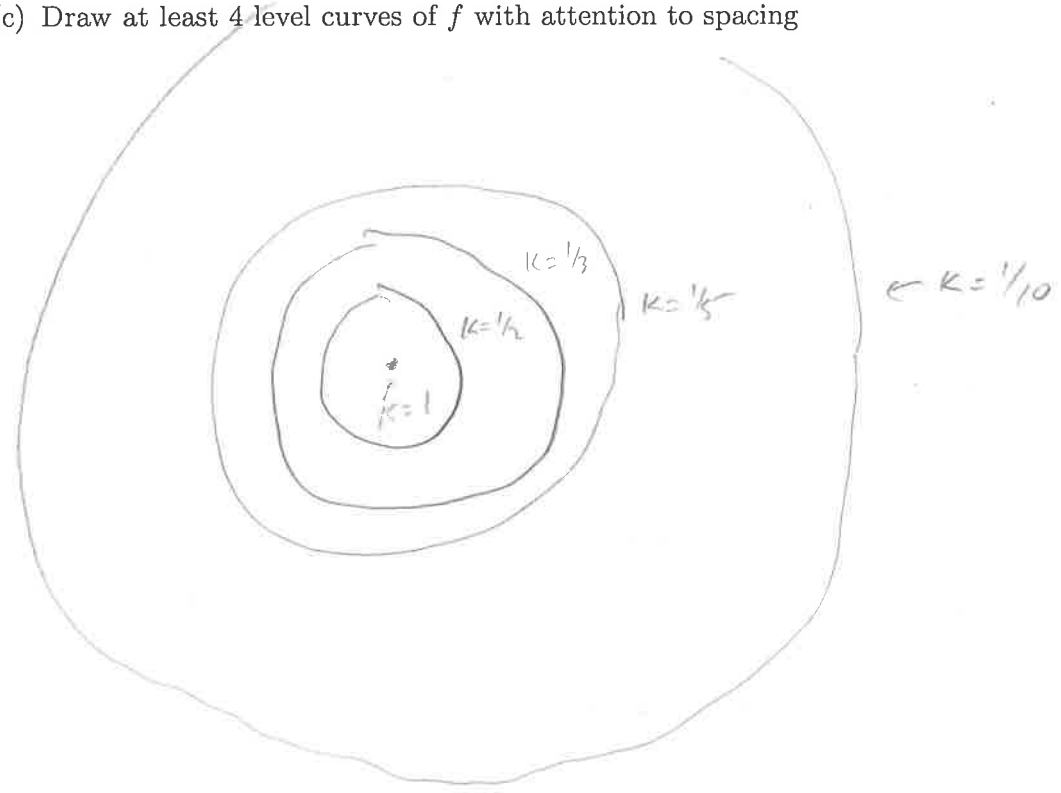
Circles

$$x^2+y^2+1 = \frac{1}{k}.$$

$$x^2+y^2 = \frac{1}{k} - 1$$

$0 < k \leq 1$

(c) Draw at least 4 level curves of f with attention to spacing



4. Find

$$\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2}.$$

(Hint: Convert x and y to polar coordinates, then simplify your fraction and analyze.)

$\lim_{r\to 0} \frac{r^3\cos^3\theta + r^3\sin^3\theta}{r^2} = r(\cos^3\theta + \sin^3\theta) < 2r \to 0$

↑ Bound!

5. Suppose a hiker is on a mountain whose elevation function can be approximated by

$$f(x,y) = 1800 - x^2 + 6xy - y^2$$

(a) If the hiker is at the point (1,-2), and they move in the positive x-direction, will they go uphill or downhill? Explain

$f_x = -2x+6y\big|_{(1,-2)} < 0$ downh.

$f_y = -2y+6x\big|_{(1,-2)} > 0$ uph.

(b) If the hiker is at the point (1,-2), and they move in the positive y-direction, will they go uphill or downhill? Explain

6. (Bonus) Clairaut's Theorem says that among its four second partial derivatives, $f(x,y)$ really has only three different expressions.

(a) How many third partial derivatives are there, and how many different expressions are there?

(b) How many nth partial derivatives are there, and how many different expressions are there?