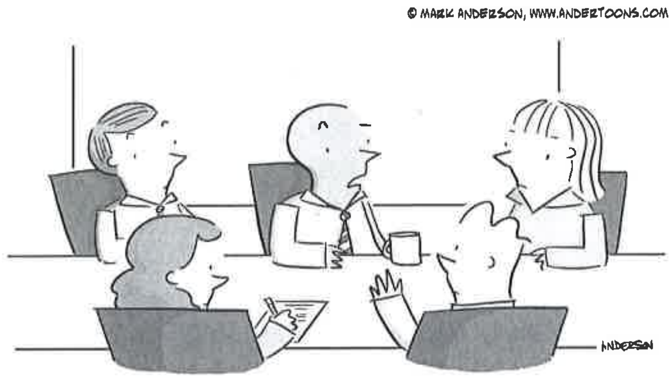
KEY

Math 225: Quiz the Seventh
November 5, 2021

You have the remainder of the period to complete this quiz. Please justify your answers where appropriate and READ ALL DIRECTIONS CAREFULLY. You may use a calculator for computation only.



"Technically, Daylight Saving Time isn't time travel, but, sure, I guess if you see another you, try to avoid him."

1. Find and classify the critical points of $f(x,y) = x^3 - 12x - y^3 + 3y$. (There are 4)

$f_x = 3x^2 - 12 = 0 \rightarrow x = \pm 2$

$f_y = -3y^2 + 3 = 0 \rightarrow y = \pm 1$

c.p. = $(2, 1)$ s.p. $(-2, 1)$ max.

$(2, -1)$ min. $(-2, -1)$ s.p.

$D = f_{xx} f_{yy} - (f_{xy})^2$

$= 6x(-6y) - 0 = -36xy$

$D_{(2,1)} = -36(2)(1) < 0 \rightarrow$ s.p.

$D(2,-1) = -36(2)(-1) \rightarrow$ max or min

$f_{xx} = 6x > 0 \rightarrow$ conc up $\rightarrow$ min

$D_{(-2,1)} = -36(-2)(1) > 0$ max or min

$f_{xx} < 0$

$D_{(-2,-1)} = -36(-2)(-1) < 0$ saddlepoint

2. Find the maximum and minimum values of $f(x,y) = x^2 - 2x + y^2$ on the circular disk $x^2 + y^2 \leq 4$.

$f(x,y) = x^2 - 2x + y^2$

$f_x = 2x - 2 = 0$ cp. $(1,0)$ $f(1,0) = \underline{-1}$ ↑ min

$f_y = 2y = 0$

on the ~~b~~oundary: $x^2+y^2 = 4$

$f = (x^2+y^2) - 2x = 4 - 2x$ $-2 \leq x \leq 2.$

$f' = -2 \neq 0$, so check endpoints!

$f(-2,0) = (-2)^2 - 2(-2) + 0 = 8$ ← max

$f(2,0) = 2^2 - 2(2) + 0 = 0$

max $= 8$ min $= -1$

3. Find the maximum volume of a can that has surface area 54π square inches. You may use any method that you wish.

$V = \pi r^2 h$

$SA = 2\pi r^2 + 2\pi r h = 54\pi$

$\hookrightarrow 2r^2 + 2rh = 54$

$V_r = 2\pi r h = \lambda(4r + 2h)$

$V_h = \pi r^2 = \lambda(2r)$

$\hookrightarrow \pi r^2 = 2\lambda r$

$\lambda = \frac{\pi r}{2}$

$2\pi r h = \frac{\pi r}{2}(4r + 2h)$

$2\pi r h = 2\pi r^2 + \pi r h$

$\pi r h = 2\pi r^2$

$h = 2r \rightarrow$

$2\pi r^2 + 2\pi r(2r) = 54\pi$

$6\pi r^2 = 54\pi$

$r^2 = 9$

$r = 3, \quad h = 6$

(The can is "squarish")

$V = \pi r^2 h = 54\pi$ ☺

4. Let $f(x,y) = x^2 + kxy + y^2$

(a) For which values of k does f have a local minimum at $(0,0)$?

$f_x = 2x + ky = 0$

$f_y = kx + 2y = 0$

$D = f_{xx} f_{yy} - (f_{xy})^2$

$(2)(2) - (k^2) = 4 - k^2 > 0 \Rightarrow |k| < 2$

$-2 < k < 2$.

$k \in (-2, 2)$

(b) For which values k does f have a local maximum at $(0,0)$

(none)

In these cases;

$f_{xx} > 0 \Rightarrow f$ is concave up

$\Rightarrow f$ has a minimum

never a maximum.

(c) For which values k does f have a saddle point at $(0,0)$?

$4 - k^2 < 0 \Rightarrow$

$k > 2$ or $k < -2$.

(d) For which k do we require further investigation at $(0,0)$

when $k = \pm 2$

(e) **Bonus**: Carry out the investigation by looking more closely at f_x and f_y at the k values you found in the last part of the last question.

if $k = 2 \rightarrow x^2 + 2xy + y^2 = f = (x+y)^2$

$f_x = 2x + 2y = 0$
$f_y = 2x + 2y = 0$ $\rightarrow x + y = 0$ minima whenever $x = -y$

$k = -2 \rightarrow x^2 - 2xy + y^2$

$f = (x-y)^2$ minima whenever $x = y$