Math 225: Practice Test 1

1. Let $x = t^2$ and $y = \frac{1}{t^2+1}$ be parametric equations.
   (a) What is the range of x and y values for the graph of these equations?
   (b) Determine the equation of the tangent line at the point $(4, \frac{1}{5})$
   (c) Determine the area bound by the axes and the line $x = 4$.
   (d) How is the graph of these equations different from the graph of $y = \frac{1}{x+1}$?
2. Plot the polar curve $r = \cos(3\theta)$ for $0 \leq \theta \leq \pi$.
   (a) Argue that there are three distinct tangent lines at the origin, and find them.
   (b) Set up the integral to find the area inside one loop of this graph
3. Argue algebraically that a sphere (of sufficiently large radius) intersects each of the coordinate planes (xy-, yz, and xz-plane) in a circle.
4. Argue, using vectors, that the diagonals of a rhombus intersect each other at right angles. (Recall that a rhombus is a parallelogram with all sides equal).
5. Consider the points $P = (2, 1, 1), Q = (3, -1, -2)$, and $R = (1, 3, 6)$
   (a) Find the equations of the lines through P and Q, through Q and R, and through R and S.
   (b) Find the three angles of the triangle PQR.
   (c) Find the equation of the plane that contains this triangle.

6. Below are a set of several equations in various 3 dimensional coordinate systems. Which of these equations represent the same surfaces? (By same, I mean the exact same surface in the same location). There are 9 equations, with three pairs and one triplet, each with the same surface.

   (a) $\sqrt{x^2 + y^2} = z$

   (b) $r = 2\sin(\theta)$

   (c) $x + y + z = 4$

   (d) $z = 2$

   (e) $\rho = \frac{2\sin(\theta)}{\sin(\phi)}$

   (f) $x^2 + (y-1)^2 = 1$

   (g) $\phi = \frac{\pi}{4}$

   (h) $r = \frac{4-z}{\sin(\theta)+\cos(\theta)}$

   (i) $\rho = 2\sec\phi$

7. The curves $\mathbf{u}(t) = \langle t, t^2, t^3 \rangle$ and $\mathbf{v}(t) = \langle \sin(t), \cos(t) - 1, t \rangle$ both intersect at the origin. Find the angle at which they intersect (hint: look at their tangent vectors).