Math 225: Practice Test 1

1. How are the level curves of the function $f(x, y) = x - y^2$ different from the level curves of the function $f(x, y) = y - x^2$. Describe both sets of level curves in detail. What do the corresponding surfaces look like?

2. Find
$$\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2 + y^2}$$
or explain why it doesn't exist.

3. Find
$$\lim_{(x,y)\to(1,1)} \frac{x^3 - y^3}{x - y}$$
or explain why it doesn't exist.

4. Verify Clairaut's Theorem in the case of $f(x, y) = \cos(x^2 y)$. Does this hold for all points (x, y)? Does the theorem hold for $\sqrt{x^2 y}$ as well?

5. Find the tangent plane to the function $f(x, y) = \frac{x}{y}$ at the point (1,2), and use it to approximate $f(.95, 2.02)$. How far is your approximation from the actual answer? *(Note: calculator needed)*

6. The temperature on a rectangular floor is given by $T(x, y) = 6x^2 - 5xy + 3y^2 - x - y$, where $0 \leq x \leq 5$ and $0 \leq y \leq 3$.

   (a) Find the maximum and minimum temperatures on the floor.

   (b) A bug is crawling at the point (2,1) and is averse to heat. Hence, he crawls in the direction in which the floor gets cold the fastest. Find the direction he is crawling in, and how fast the temperature is changing in that direction.

7. Someone tells you that the partial derivatives of a continuous function $f(x, y)$ are $f_x = xy$ and $f_y = \frac{x^2}{2} + x^3$. Should you believe them? If they are right about f_x, what must f_y look like?

8. Maximize $f(x, y) = x^2 y$ subject to the constraint $x^2 + y^2 = 1$.

9. Describe how you would estimate $\iint_R \frac{x}{y}\, dA$, where R is the rectangle $[0,1] \times [1,3]$ using $m = n = 4$ subdivisions and upper left-hand corners as your sample points (include a list of values you'd calculate). Then find the exact area by integration.

10. Find the volume under the surface $z = \frac{x}{(y+1)^2}$ and above the region in the xy-plane bounded by the parabola $y = x^2$ and the line $y = 4x$.

11. Reverse the order of integration on $\int_{-1}^{1} \int_{y-3}^{y^2} f(x,y) dx dy$

12. Determine $\iint_R \frac{y}{x}\, dA$, where A is the region bound by $y = 0, y = x, x^2 + y^2 = 2$, and $x^2 + y^2 = 25$.

Note: The test has too much information on it already to have a question from section 15.5, so please only focus your studies up through section 15.4.