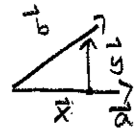
Exam Review

2) vector projections : want to project $\vec{b}$ on to $\vec{a}$

$\vec{a} = \langle 2, 1, -1 \rangle$

$\vec{b} = \langle 3, 0, 1 \rangle$



Is $\vec{y} \perp \vec{a}$?

$$\underbrace{\langle 2, 1, -1 \rangle}_{\vec{a}} \cdot \underbrace{\langle \tfrac{4}{3}, -\tfrac{5}{6}, \tfrac{11}{6} \rangle}_{\vec{y}} = 0$$

$$\text{proj}_{\vec{a}} \vec{b} = \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|^2} \cdot \vec{a}$$

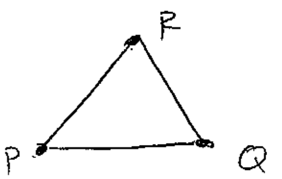
$$= \frac{5}{6} \langle 2, 1, -1 \rangle = \langle \tfrac{5}{3}, \tfrac{5}{6}, -\tfrac{5}{6} \rangle$$

$$\langle 3, 0, 1 \rangle = \underbrace{\langle \tfrac{5}{3}, \tfrac{5}{6}, -\tfrac{5}{6} \rangle}_{\vec{x}} + \underbrace{\langle \tfrac{4}{3}, -\tfrac{5}{6}, \tfrac{11}{6} \rangle}_{\vec{y}}$$

$\vec{y} = \vec{b} - \vec{x}$

---

4) $P = (1, 3, -2)$ $\quad Q = (3, 1, 6)$ $\quad R = (1, 2, -1)$



$\vec{PQ} = \langle 2, -2, 8 \rangle \quad |\vec{PQ}| = \sqrt{72}$

$\vec{QR} = \langle -2, 1, -7 \rangle \quad |\vec{QR}| = \sqrt{54} = 3\sqrt{6}$

$\vec{PR} = \langle 0, -1, 1 \rangle \quad |\vec{PR}| = \sqrt{2}$

Parimeter $= \sqrt{72} + \sqrt{54} + \sqrt{2}$

Use... $\vec{RQ} \cdot \vec{PQ}$ to find $\measuredangle Q = \cos^{-1}\left(\frac{62}{\sqrt{72} \cdot \sqrt{54}} \right)$

$\vec{RP} \cdot \vec{RQ}$ to find $\measuredangle R = \cos^{-1}\left(\frac{-8}{\sqrt{54} \cdot \sqrt{2}} \right) \longleftarrow$ obtuse $\measuredangle$: $\cos\theta$ is less than 0

$\vec{PR}$ $\vec{PQ}$ to fin $\measuredangle P = \cos^{-1}\left(\frac{10}{\sqrt{72} \cdot \sqrt{2}} \right)$

To find normal vector... take cross product of two vectors

$$\begin{array}{r} \vec{RQ} = \langle 2, -1, 7 \rangle \\ \times \; \vec{RP} = \langle 0, 1, -1 \rangle \\ \hline \langle -6, 2, 2 \rangle \end{array}$$

Plug in other point...

$-6(x-1) + 2(y-3) + 2(z+2) = 0$

Equation of plane: $\boxed{-6 + 2y + 2z = 4}$

---

6)

Cylindrical

$x \longrightarrow x = r\cos\theta$

$y \longrightarrow y = r\sin\theta$

$z \longrightarrow z = z$

$\theta = \arctan\left(\frac{y}{x}\right)$

$r = \sqrt{x^2 + y^2}$

$\longrightarrow$

Spherical

$x = \rho \sin\phi \cos\theta$

$y = \rho \sin\phi \sin\theta$

$z = \rho\cos\phi$

$r = \rho\sin\phi$

$\rho = \sqrt{x^2 + y^2 + z^2}$

$\theta = \arctan\left(\frac{y}{x}\right)$

$\phi = \arcsin \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}$

$\phi = \arccos\left(\frac{z}{\sqrt{x^2+y^2+z^2}} \right)$

6) a) $x^2+y^2-9z^2=1$

cylindrical coordinates $\Rightarrow r^2-9z^2=1$

#7) d) $\rho^2(\sin^2\phi - 9\cos^2 \phi)=1$

b) $y=3$

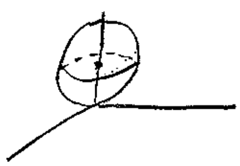
e) $r=3\csc\theta$

$\hookrightarrow 3 = r\sin\theta$

g) $\rho = 3\csc\phi\csc\theta$

$3=\rho\sin\phi\sin\theta$

plane: $\vec{n}=\langle 0,1,0\rangle$



c) $x=y$ $\quad \vec{n}=\langle 1,-1,0\rangle$

$x-y=0$

h) $\theta=\frac{\pi}{4}$

plane, $\vec{n}=\langle 1,-1,0\rangle$

f) $\rho = 3\cos\theta$

i) $x^2+y^2+(z-\frac{3}{2})^2=\frac{9}{4}$

$x^2+y^2+z^2-3z+\cancel{\frac{9}{4}}=\cancel{\frac{9}{4}}$

$\rho^2=3\rho\cos\phi$

$ax+by+cz=d$ (plane) $\to$ normal vector $(\vec{n})$ to plane is $\langle a,b,c\rangle$

In order to get normal vector one has to peel off coefficients.

★ Know how to get normal vector and plane.

9) A) $r(t)=\langle 3\cos t, 2\sin t, t\rangle$ $\quad r'(t)=\langle -3\sin t, 2\cos t, 1\rangle$

line: $\vec{r}(\frac{\pi}{2})+t\vec{r}'(\frac{\pi}{2})$ $\quad r'(\frac{\pi}{2})=\langle -3,0,1\rangle$

$r(\frac{\pi}{2})=\boxed{\langle 0,2,\frac{\pi}{2}\rangle + t\langle -3,0,1\rangle}$

B) $\vec{r}(t)=\langle 3\cos t, 2\sin t, t\rangle_{(t=\pi)}$ $\quad (-3,0,\pi)$

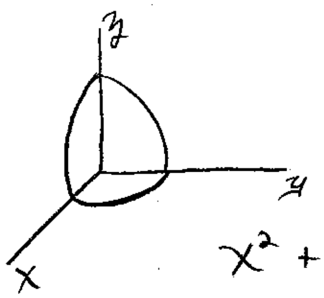
$\vec{s}(t)=\langle t-3, t^2, t^3+\pi\rangle$ $(t=0)$

$r'(t)=\langle -3\cos t, 2\cos t, 1\rangle \to \langle 0,-2,1\rangle$

$s'(t)=\langle 1, 2t, 3t^2\rangle \to \cdot\langle 1,0,0\rangle$

$0+0+0=0$ $\quad \boxed{\measuredangle = \frac{\pi}{2}}$

5.)

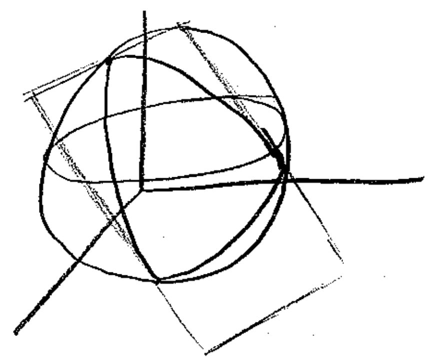


$$x^2+y^2+z^2>4$$

Rectangular	Spherical
$x>0$	$\rho>2$
$y>0$	$0<\phi<\frac{\pi}{2}$
$z>0$	$0<\theta<\frac{\pi}{2}$

8.) $x^2+y^2+z^2=4$

$x+y=2$



$x=t$

$y=2-t$

$z^2=4-t^2-(2-t)^2$

$z=\sqrt{4-t^2-(2-t)^2}$

$-2\le x\le 2$

$-2\le y\le 2$

$-2\le z\le 2$

10.) $r(t)=\langle 3\cos t, 2\sin t, t\rangle$

$\vec{r}'(t)=\langle -3\sin t, 2\cos t, 1\rangle$

$$\int_0^{2\pi}\sqrt{9\sin^2 t+4\cos^2 t+1}\,dt$$

$$\int_0^{2\pi}\sqrt{5\sin^2 t+5}\,dt$$

Exam Review

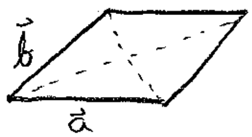
1.) Two $\parallel$ lines have direction vectors which differ by a Scalar Multiple.

Ex:] $\vec{r}_1(t) = \langle 1, 0, 2 \rangle + t\langle 1, 1, 2 \rangle$

$\vec{r}_2(t) = \langle 2, 0, 5 \rangle + t\langle -3, -3, -6 \rangle$

$\vec{r}_3(t) = \langle 1, 4, 6 \rangle + t\langle 2, 2, 2 \rangle$

$\vec{r}_1 \parallel \vec{r}_2$ but $\vec{r}_2 \not\parallel \vec{r}_3$

3.) 

Diagonals: $\vec{a} + \vec{b}$; $\vec{a} - \vec{b}$

If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ then $\vec{a} \perp \vec{b}$.

$$|\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2$$

$$(\vec{a} + \vec{b})(\vec{a} + \vec{b}) = (\vec{a} - \vec{b})(a - \vec{b})$$

$$\cancel{|\vec{a}|^2} + 2\vec{a}\vec{b} + \cancel{|\vec{b}|^2} = \cancel{|\vec{a}|^2} - 2\vec{a}\vec{b} + \cancel{|\vec{b}|^2}$$

$$4\vec{a}\vec{b} = 0 \quad \Rightarrow \quad \vec{a} \perp \vec{b}.$$