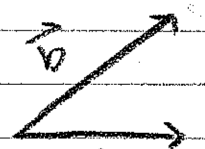
Practice Exam #1

1. Yes, but only up to a scalar multiple. Parallel lines have the "same" direction vectors.

$\vec{r}_1(t) = \langle 0,1,2\rangle + t\langle 1,1,2\rangle$
$\vec{r}_2(t) = \langle 1,2,5\rangle + t\langle -3,-3,-6\rangle$
$\vec{r}_3(t) = \langle 5,-1,0\rangle + t\langle 2,2,2\rangle$

$\vec{r}_1 \parallel \vec{r}_2$ but $\vec{r}_2 \nparallel \vec{r}_3$

2.



$\langle 2,1,-1\rangle$

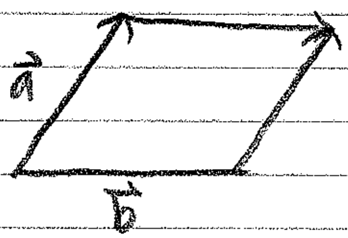
$$\langle 3,0,1\rangle = \left\langle \frac{5}{3}, \frac{5}{6}, \frac{-5}{6}\right\rangle + \left\langle \frac{4}{5}, \frac{-5}{6}, \frac{11}{6}\right\rangle$$
$$\vec{b} = \vec{x} + \vec{y}$$

$$\text{Proj}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}(\vec{a})$$

$$\frac{5}{6}\langle 2,1,-1\rangle$$

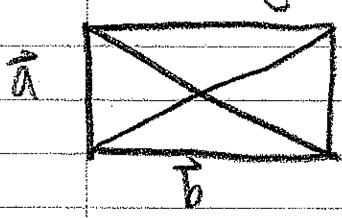
$$\vec{y}\cdot\vec{a} = \left\langle \frac{4}{3}, \frac{-5}{6}, \frac{11}{6}\right\rangle$$
$$\langle 2,1,-1\rangle$$
$$\frac{8}{3} - \frac{5}{6} - \frac{11}{6} = 0$$
$$\vec{y} \perp \vec{a}$$

3.



diagonals: $\vec{a}-\vec{b}$ and $\vec{a}+\vec{b}$

Rectangle



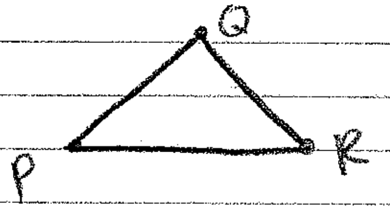
$$|\vec{a}-\vec{b}|^2 = |\vec{a}+\vec{b}|^2 \iff \vec{a}\perp\vec{b}$$

$$(\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b}) = (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b})$$
$$\cancel{|\vec{a}|^2} + 2(\vec{a}\cdot\vec{b}) + \cancel{|\vec{b}|^2} = \cancel{|\vec{a}|^2} + 2(\vec{a}\cdot\vec{b}) + \cancel{|\vec{b}|^2}$$

$$4(\vec{a}\cdot\vec{b}) = 0$$
$$\vec{a}\cdot\vec{b} = 0$$
$$\vec{a}\perp\vec{b}$$

hi megan + rachel!

4. $P(1,3,-2)$ $Q(3,1,6)$ $R(1,2,-1)$



$\overrightarrow{PQ} = \langle 2,-2,8\rangle$ magnitude $= \sqrt{72} \Leftrightarrow \overrightarrow{QP} = \langle -2,2,8\rangle$

$\overrightarrow{PR} = \langle 0,-1,1\rangle$ magn $= \sqrt{2} \Leftrightarrow \overrightarrow{RP} = \langle 0,1,-1\rangle$

$\overrightarrow{QR} = \langle -2,1,-7\rangle$ $m = \sqrt{54} \Leftrightarrow \overrightarrow{RQ} = \langle 2,-1,7\rangle$

$\measuredangle P = \cos^{-1}\left(\dfrac{10}{\sqrt{72}\sqrt{2}}\right)$ perimeter $= \sqrt{72}+\sqrt{2}+\sqrt{54}$

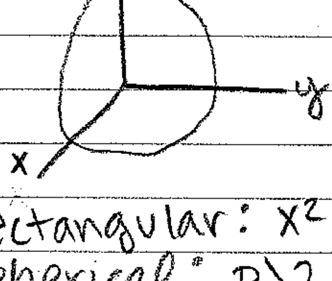
$\measuredangle Q = \cos^{-1}\left(\dfrac{62}{\sqrt{72}\cdot\sqrt{54}}\right)$

$\measuredangle R = \cos^{-1}\left(\dfrac{-8}{\sqrt{2}\sqrt{54}}\right)$ ← dot product is negative
$\Rightarrow$ angle is obtuse
$\Rightarrow \Delta PQR$ is obtuse

Plane: $\langle 0,1,-1\rangle$
$\langle 2,-1,7\rangle$
$\langle 6,-2,-2\rangle$

$6(x-1)-2(y-2)-2(z+1)=0$ Equation for plane

5.



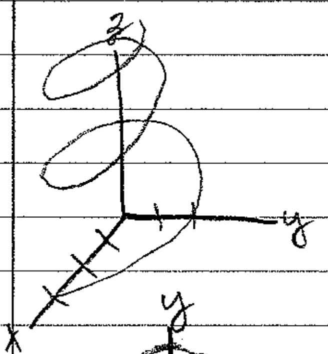
$x>0$
$y>0$
$z>0$

$x^2+y^2+z^2>4$

rectangular: $x^2+y^2+z^2>4$

Spherical: $p>2$, $0<\theta<\pi/2$, $0<\phi<\pi/2$

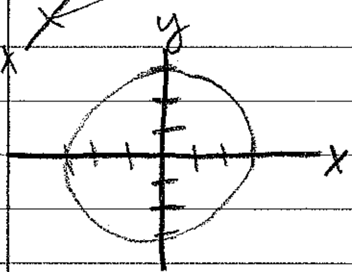
9. $r(t) = \langle 3\cos(t), 2\sin(t), t \rangle$



a) $t = \pi/2$

$\vec{r}(\pi/2) + t\,r'(\pi/2)$

$\vec{l}(t): \langle 0, 2, \frac{\pi}{2} \rangle + t\langle -3, 0, 1 \rangle$



b) $\vec{s}(t) = \langle t-3, t^2, t^3+\pi \rangle$ at $(-3, 0, \pi)$

$s'(t) = \langle 1, 2t, 3t^2 \rangle$ $t=0$

$s'(0) = \langle 1, 0, 0 \rangle$

$\langle 0, -2, 1 \rangle \cdot \langle 1, 0, 0 \rangle = 0$, so angle $\theta = \pi/2$

c) $\int_0^{2\pi} \sqrt{9\sin^2 t + 4\cos^2 t + 1}$

$= \int_0^{2\pi} \sqrt{5\sin^2 t + 5}\, dt$

d) $t = \pi$ $\vec{r}(\pi) = \langle -3, 0, \pi \rangle$

$T = \langle 0, \frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \rangle$
x

$N = \langle 1, 0, 0 \rangle$

$B = \langle 0, \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \rangle$

normal plane: $0(x+3) - \frac{2}{\sqrt{5}}(y-0) + \frac{1}{\sqrt{5}}(z-\pi) = 0$
(T)

osculating plane: $0(x+3) + \frac{1}{\sqrt{5}}(y-0) + \frac{2}{\sqrt{5}}(z-\pi) = 0$
(B)

Sarah is cool