Math 225: Practice Exam the First

This practice test is meant to represent the types of topics that you will encounter on Friday's Exam.

1. Can parallel lines have different direction vectors? Explain.

2. Project the vector $\mathbf{b} = \langle 3, 0, 1 \rangle$ onto the vector $\mathbf{a} = \langle 2, 1, -1 \rangle$. Use this to express the vector $\mathbf{b} = \mathbf{x} + \mathbf{y}$, where $\mathbf{x}$ is parallel to $\mathbf{a}$ and $\mathbf{y}$ is perpendicular to $\mathbf{a}$.

3. Argue (using vectors) that a parallelogram is a rectangle if and only if its diagonals are equal in length.

4. Consider the points $P = (1, 3, -2), Q = (3, 1, 6)$, and $R = (1, 2, -1)$
   (a) Find the perimeter of triangle PQR
   (b) Is this triangle right, acute, or obtuse?
   (c) Find the equation of the plane that contains this triangle.

5. (a) Describe, using inequalities and rectangular coordinates, the points in the first octant outside of the sphere of radius 2 centered at the origin.
   (b) Same as part (a), but with spherical coordinates.

6. Below are a set of several equations in various 3 dimensional coordinate systems. Which of these equations represent the same surfaces? (By same, I mean the exact same surface in the same location). There are 9 equations, with three pairs and one triplet, each with the same surface.
   (a) $x^2 + y^2 - 9z^2 = 1$
   (b) $y = 3$
   (c) $x = y$
   (d) $\rho^2(\sin^2(\phi) - 9\cos^2\phi) = 1$
   (e) $r = 3\csc(\theta)$
   (f) $\rho = 3\cos(\phi)$
   (g) $\rho = 3\csc(\phi)\csc(\theta)$

(h) $\theta = \frac{\pi}{4}$

(i) $x^2 + y^2 + (z - \frac{3}{2})^2 = \frac{9}{4}$

7. Which of the equations in Question 6 are planes? For those which are planes, give the normal vector. Which are spheres? For those which are spheres, give the center and the radius.

8. Parametrize the curve of intersection between the sphere $x^2+y^2+z^2 = 4$ and the plane $x + y = 2$. Describe the curve in words.

9. Consider the elliptical helix given by the vector valued function $\mathbf{r}(t) = \langle 3\cos(t), 2\sin(t), t\rangle$.

   (a) Find the tangent line to this curve when $t = \frac{\pi}{2}$

   (b) Find the angle between this curve and the curve $\mathbf{s}(\mathbf{t}) = \langle t - 3, t^2, t^3 + \pi\rangle$ at the point $(-3, 0, \pi)$.

   (c) Set up, but do not compute, the integral which gives the length of this curve from $t = 0$ to $t = 2\pi$.

   (d) At the point when $t = \pi$, the curve has the unit Tangent vector $\langle 0, \frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\rangle$ and unit Normal vector $\langle 1, 0, 0\rangle$. Calculate the Binormal vector, the normal plane, and the osculating plane at this point.