Math 225: Exam the Second

November 8, 2006

You have 90 minutes to complete this closed-book, closed-notes, and closed-colleague exam. You may use a calculator but be prepared to justify your answers if you do so. **READ ALL QUESTIONS CAREFULLY**, as I am more lenient with partial credit if I feel you've done so.

1. Consider the function $f(x, y) = e^{x+y}$.
   (a) For which values k can one draw level curves of the form $f(x, y) = k$?
   (b) Draw enough of the level curves to describe the behavior of the function as x and y both get large.

2. Find
$$\lim_{(x,y)\to(1,1)} \frac{(x-y)}{\sqrt{x}-\sqrt{y}}$$
   or explain why it doesn't exist.

3. Which of the following functions have $f_{xy} = 0$? Note: More than one answer is possible. Don't spend too much time on this one.
   (a) $f(x, y) = x\cos(x^2) + y\arcsin(y^2)$
   (b) $f(x, y) = y\cos(x^2) + x\arcsin(y^2)$
   (c) $f(x, y) = x^{\cos(x)} - ye^{sin(y)}$
   (d) $f(x, y) = \cos(x^2y^3)$

4. Let $f(x, y) = \sqrt{x^2 + y^3}$.
   (a) Estimate $f(1.04, 1.98)$.
   (b) Find the directional derivative of f at the point $(1, 2)$ in the direction of $\mathbf{i} + \mathbf{j}$
   (c) Find the gradient and the maximum rate of change at the point (1,2).

5. Let $f(x, y, z) = z(x^2 + y^2)$.
   (a) Compute the partial derivatives f_x, f_y and f_z.
   (b) Convert the equation to cylindrical coordinates and compute f_r. Verify your answer using the chain rule.
   (c) Convert the equation to spherical coordinates and compute f_ρ. Verify your answer using the chain rule.

6. Consider the function $f(x, y) = x^2 + y^2 + kxy$ for a constant k.
   (a) For which values k does $f(x, y)$ have a local minimum at $(0, 0)$?
   (b) For which values k does $f(x, y)$ has a saddle point at $(0, 0)$?
   (c) For which values k do we cry at $(0, 0)$?

7. Using Lagrange Multipliers, find the volume of the largest rectangular box that can fit in the first octant and under the plane $ax + by + cz = d$, where a, b, c and d are positive constants.

8. Find the volume bound by the coordinate planes, the planes $x = 1$, $y = 2$ and the paraboloid $z = 9 - x^2 - y^2$.

9. Reverse the order of integration on

$$\int_0^4 \int_{\frac{y}{2}}^2 \cos(x^2)\, dx\, dy$$

and find its value.