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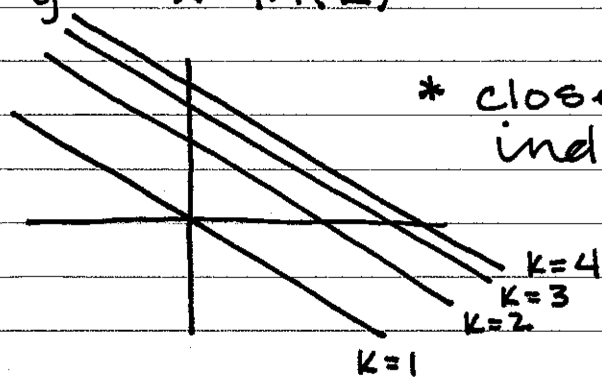
Exam the Second

1. $f(x,y) = e^{x+y}$

a. $k = e^{x+y} \quad k > 0$

b. $y = -x + \ln(k)$



* closer level curves indicate a steeper surface.

2. $$\lim_{(x,y)\to(1,1)} \frac{x-y}{\sqrt{x}-\sqrt{y}} \cdot \frac{(\sqrt{x}+\sqrt{y})}{\sqrt{x}-\sqrt{y}} = \frac{\cancel{(x-y)}(\sqrt{x}+\sqrt{y})}{\cancel{\sqrt{x}^2-\sqrt{y}^2}} = 2$$

3. $f_{xy} = 0$

$(f_x)_y \quad f_x$ all x's
$(f_y)_x \quad f_y$ all y's

a. $f(x,y) = x\cos(x^2) + y\arcsin(y^2)$
$f_{xy} = 0$

c. $f(x,y) = x^{\cos(x)} - ye^{\sin(y)}$
$f_{xy} = 0$

HI SARAH!

4. $f(x,y) = \sqrt{x^2+y^3} = (x^2+y^3)^{1/2}$

a. $f(1.04, 1.98)$

$\downarrow$

$f(1,2) = 3$ $\qquad f_x = \frac{1}{2}(x^2+y^3)^{-\frac{1}{2}} 2x$

$f_x(1,2) = 1/3$ $\qquad f_y = \frac{1}{2}(x^2+y^3)^{-\frac{1}{2}} \cdot 3y^2$

$f_y(1,2) = 2$

$f(1.04, 1.98) \approx f(1,2) + f_x(1,2)(1.04-1) + f_y(1,2)(1.98-2)$

$\approx 3 + 1/3 \; (.04) + 2 \; (-.02)$

$f(1.04, 1.98) \approx 3 - 8/300$

b. $D_{\vec{u}}f = \vec{\nabla} f \cdot \vec{u}$

$\vec{\nabla} f = \langle 1/3, 2 \rangle$ $\qquad D_{\vec{u}}f = \frac{7\sqrt{2}}{6}$

$\vec{u} = \langle \sqrt{2}/2, \sqrt{2}/2 \rangle$

c. $\vec{\nabla} f = \langle 1/3, 2 \rangle$

The maximum rate of change is equal to the magnitude of the gradient.

$|\vec{\nabla} f| = \frac{\sqrt{37}}{3}$

5. $f(x,y,z) = z(x^2+y^2)$

a. $f_x = z(2x)$
$f_y = z(2y)$
$f_z = (x^2+y^2)$

b. $f(r,\theta,z) = zr^2$

$f_r = 2rz$
$f_\theta = \theta$
$f_z = r^2$

$$f_\theta = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial \theta} + \frac{\partial f}{\partial z}\cdot\frac{\partial z}{\partial \theta}$$
$$f_\theta = (2zx)(-r\sin\theta) + (2zy)(r\cos\theta) + (x^2+y^2)(0)$$
$$f_\theta = -2zr^2\cos\theta\sin\theta + 2zr^2\sin\theta\cos\theta$$
$$f_\theta = 0$$

c. $f(\rho,\theta,\phi) = \rho^3\cos\phi\sin^2\phi$

$f_\rho = 3\rho^2\cos\phi\sin^2\phi$
$f_\theta = 0$
$f_\phi = \rho^3[-\sin^3\phi + 2\cos^2\phi\sin\phi]$

6. $f(x,y) = x^2 + y^2 + kxy$

$f_x = 2x + ky$
$f_y = 2y + kx$
$f_{xx} = 2$
$f_{yy} = 2$
$f_{xy} = k$

$D = 4 - k^2 \to$
max/min: $D > 0$
$(2 > k > -2)$
saddle: $D < 0$
$(k > 2 \text{ or } k < -2)$
inconclusive: $D = 0$
$(k = \pm 2)$

7. $ax + by + cz = d \qquad V = xyz$

$\vec{\nabla} f = \lambda \vec{\nabla} g$

$V_x = yz \cdot x = \lambda a x$
$V_y = xz \cdot y = \lambda b y$
$V_z = xy \cdot z = \lambda c z$

$\lambda a x = \lambda b y = \lambda c z$
$ax = by = cz$

$x = \dfrac{d}{3a}$
$y = \dfrac{d}{3b}$
$z = \dfrac{d}{3c}$

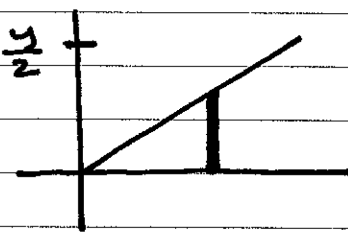
$$\boxed{V_T = \frac{d^3}{27abc}}$$

8. $z = 9 - x^2 - y^2 \qquad [0,1] \times [0,2]$

$$\int_0^1 \int_0^2 9 - x^2 - y^2 \, dy\,dx$$

* can switch order of integration

9. $$\int_0^4 \int_{\frac{y}{2}}^2 \cos(x^2) \, dx\,dy$$



$$\int_0^2 \int_0^{2x} \cos(x^2) \, dy\,dx$$

$$= \int_0^2 2x \cos(x^2) \, dx$$

$$= \sin(x^2) \Big|_0^2$$

$$= \boxed{\sin 4}$$