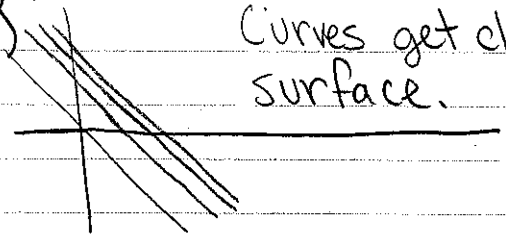
1) $f(x,y)=e^{x+y}$

$K=e^{x+y} \rightarrow \ln K = x+y \rightarrow y=-x+\ln K$

a) $K>0$

b)



Curves get closer and closer, indicating a steeper surface.

2) $\lim_{(x,y)\to(1,1)} \frac{(x-y)}{\sqrt{x}-\sqrt{y}} \cdot \frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}} = \frac{\cancel{(x-y)}\sqrt{x}+\sqrt{y}}{\cancel{\sqrt{x}^2-\sqrt{y}^2}} = 2 \rightarrow$ Shows the limit exists.

To show the limit doesn't exist find two different limits from two different directions.

3) f_x must be all x's, or f_y must be all y's.

a) yes b) no c) yes d) no

4) $f(x,y)=\sqrt{x^2+y^3} \rightarrow (x^2+y^3)^{1/2}$

a) Estimate $f(1.04, 1.98) \rightarrow$ near $(1,2)$

$f(1,2)=3$

$f_x = \frac{1}{2}(x^2+y^3)^{-1/2} 2x \rightarrow f(1,2)=\frac{1}{3}$

$f_y = \frac{1}{2}(x^2+y^3)^{-1/2} 3y^2 \rightarrow f(1,2)=2$

*formula $\leftarrow$ $f(1.04,1.98) \approx f(1,2)+f_x(1,2)(1.04-1)+f_y(1,2)(1.98-2)$

$= 3+\frac{1}{3}(.04)+2(-.02) \approx 3-\frac{8}{300}$

b) $D_{\vec{u}} f = \vec{\nabla} f \cdot \vec{u}$ $i,j = \langle 1,1\rangle$

$\langle \frac{1}{3}, 2\rangle \cdot \langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\rangle = \frac{\sqrt{2}}{6}+\sqrt{2} = \boxed{\frac{7\sqrt{2}}{6}}$

c) $\vec{\nabla} = \langle \frac{1}{3}, 2\rangle$ maximum rate of change: $|\vec{\nabla} f| = \sqrt{\frac{1}{9}+4} = \frac{\sqrt{37}}{3}$

5) $f(x,y,z) = z(x^2+y^2)$

a) $f_x = z(2x)$

$f_y = z(2y)$

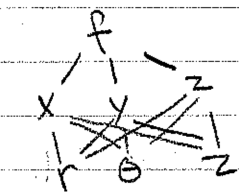
$f_z = x^2+y^2$

b) $f(r,\theta,z) = zr^2$

$f_r = 2rz$

$f_\theta = 0$

$f_z = r^2$



Verify using chain rule!, $f_\theta = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \theta} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial \theta}$ $\quad x = r\cos\theta$, $y = r\sin\theta$, $z = z$

$= (2zx)(-r\sin\theta) + (2zy)(r\cos\theta) + (x^2+y^2)(0)$

$= -2zr^2\cos\theta\sin\theta + 2zr^2\sin\theta\cos\theta = 0$

c) $f(\rho,\phi,\theta) = \rho\cos\phi(\rho^2\sin^2\phi) = \rho^3\cos\phi\sin^2\phi$

$f_\rho = 3\rho^2\cos\phi\sin^2\phi$

$f_\theta = 0$

$f_\phi = \rho^3[-\sin^3\phi + 2\cos^2\phi\sin\phi]$

6) $f(x,y) = x^2+y^2+kxy$

a) $f_x = 2x+ky$

$f_y = 2y+kx$

$f_{xx} = 2$ → If $D>0$ it will be a minimum

$f_{yy} = 2$

$f_{xy} = k$

$D = 4-k^2$
- max/min = $D>0 \rightarrow 2>k>-2$
- saddle = $D<0 \rightarrow k>2$ or $k<-2$
- ¿¿ $= D=0 \rightarrow k=\pm 2$

7) $f \rightarrow V = xyz$, $g \rightarrow ax+by+cz=d$

$\vec{\nabla} f = \lambda \vec{\nabla} g$

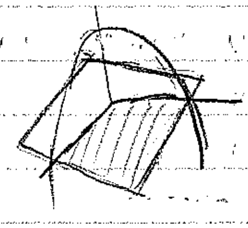
$V_x = xyz = \lambda ax$

$V_y = yxz = \lambda by$ → $\lambda ax = \lambda by = \lambda cz$ → $ax = by = cz$ → $x = \frac{d}{3a}$, $y = \frac{d}{3b}$, $y = \frac{d}{3c}$ → $x \cdot y \cdot z = \boxed{\frac{d^3}{27abc}}$

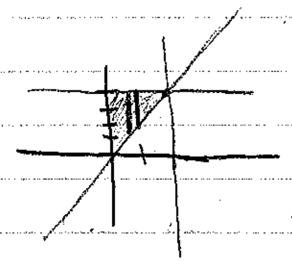
$V_z = zxy = \lambda cz$

8) $x=1,\ y=2,\ z=9-x^2-y^2$

$$\int_0^1\int_0^2 9-x^2-y^2\,dy\,dx\ \ldots$$



9) $$\int_0^4\int_{\frac{y}{2}}^2 \cos(x^2)\,dx\,dy$$



$$\int_0^2\int_0^{2x}\cos(x^2)\,dy\,dx \rightarrow \int_0^2 y\cos(x^2)\Big|_0^{2x}dx$$

$$\rightarrow \int_0^2 2x\cos(x^2)\,dx \rightarrow \sin(x^2)\Big|_0^2 = \sin 4$$

$U=x^2,\ du=2x\,dx$

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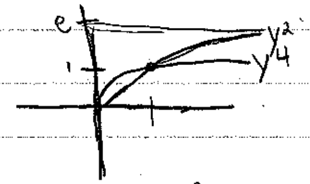
1) $$\iint_R xe^{xy}\,dA,\ R=[0,1]\times[0,1] \rightarrow \int_0^1\int_0^1 xe^{xy}\,dy\,dx$$

$$\rightarrow \int_0^1 \frac{xe^{xy}}{x}\Big|_0^1 dx \rightarrow \int_0^1 e^x-1\,dx \rightarrow e^x-x\Big|_0^1 \rightarrow e-1-e^0 = \boxed{e-2}$$

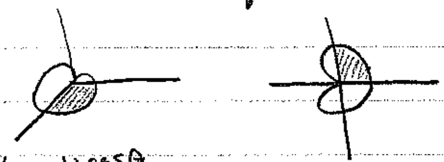
2) $$\int_1^e\int_{y^2}^{y^4}\frac{1}{x}\,dx\,dy$$



$$\rightarrow \int_1^e \ln x\Big|_{y^2}^{y^4}dy \rightarrow \int_1^e \ln y^4-\ln y^2\,dy \rightarrow \int_1^e 2\ln y\,dy \rightarrow$$

$$\rightarrow 2(y\ln y-y)\Big|_1^e = 2(e\cdot 1-e)-(0-1)=2$$

6) Find volume under $z=xy$ and inside $r=1+\cos\theta$ in the first quadrant.



$0 \le r \le 1+\cos\theta$

$0 \le \theta \le \frac{\pi}{2}$ ← restricted to first octant

$$\iint_R xy\,dA$$

$$\int_0^{\pi/2}\int_0^{1+\cos\theta} r^3\sin\theta\cos\theta\,dr\,d\theta \to \frac{1}{4}\int_0^{\pi/2}\underbrace{(1+\cos\theta)}_{u}^4\sin\theta\cos\theta\,d\theta$$

$\to$ $u=1+\cos\theta$, $du=-\sin\theta\,d\theta$ $\to -\frac{1}{4}\int_1^2 u^4(u-1)\,du$ ...

9) $\iiint_R \underbrace{x^3+xy^2}_{x(x^2+y^2)}\,dV$, R is the region in the first octant under $z=1-x^2-y^2$

$$\to \int_0^{\pi/2}\int_0^1\int_0^{1-r^2} r^4\cos\theta\,dz\,dr\,d\theta \ldots$$

$0 \le z \le 1-r^2$

$0 \le r \le 1$

$0 \le \theta \le \frac{\pi}{2}$ → because we're restricted to first octant