Math 225: Practice Final
Spring, 2008

These questions are meant to be representative of the types you'll encounter on the Final Exam. Use these along with past quizzes and exams to get an idea of the scope and depth of questions that you may see.

1. Consider $A = (1, 2, 3), B = (-1, 0, 4), C = (2, -3, 5)$
   (a) Find the equations of the lines AB and AC and the angle between these lines.
   (b) Find the equation of the plane containing AB and AC.
   (c) Find the equation of the line normal to the plane passing through the point C.

2. Consider the curve given by parametric equations:
$$x(t) = \cos(t),\ y(t) = 2\cos(t)\ z(t) = \sin(t)$$
   (a) What are the permissible ranges of x, y and z values?
   (b) What do you see when you look directly down the x-axis? The y-axis? The z-axis?
   (c) This curve is the intersection of what plane and what cylinder?
   (d) Set up, but do not compute, the integral to give the length of this curve for $0 \leq t \leq 2\pi$.

3. (a) Let $f(x, y) = \frac{y}{x^2}$. Describe the level curves of f in detail, giving shapes as well as points where f is undefined.
   (b) Does this curve have any local maxima, local minima, or saddle points? Explain why or why not.
   (c) Find the direction and rate of maximum change of f at the point (1,2).

4. Let $z = x\cos(y)$.
   (a) Find the equation of the tangent plane at $(1, \pi)$.
   (b) Use the plane to approximate $f(.99, 3.1515926...)$.
   (c) Suppose that $x = \sin(t)$ and $y = e^t$. Find $\frac{dz}{dt}$

5. What is the minimum distance between the curve $xy = 1$ and the origin?

6. Switch the order of integration on
$$\int_0^1 \int_{-\sqrt{1-y}}^{\sqrt{1-y}} xy\ dx\ dy$$
   and solve it.

7. Let V be the region bound by $z = \sqrt{x^2 + y^2}$ and $z = h$.
   (a) Set up the volume V as either a double or triple integral in rectangular coordinates.

(b) Determine the volume of V by doing a double integral in polar coordinates. Your answer should be simple and verifiable from high school geometry.

(c) Verify your answer to part (b) by doing a triple integral in spherical coordinates. (Warning: May take some time).

8. Determine
$$\iint_R x^2 - y^2 \, dA$$
where R is the square with vertices $(1,0),(0,1),(-1,0)$, and $(0,-1)$ by substituting $x = \frac{u+v}{2}$, $y = \frac{u-v}{2}$.

9. Compute two of the following three integrals.

(a)
$$\int_C x^2 y \, ds$$
where C is the upper left quarter of the unit circle centered at the origin.

(b)
$$\int_C (x+y)\, dx + x \, dy$$
where C is the parabolic curve from $(1,2)$ to $(2,5)$ given by $y = x^2 + 1$.

(c)
$$\oint_C (x^3 + 2y)\, dx + (x + \cos(y))\, dy$$
where C is the parallelogram with vertices $(0,0),(2,2),(6,2)$, and $(4,0)$, oriented counterclockwise.

10. List as many (at least 3) integral theorems as you can and draw any parallels that you can between them. Note: I want *theorems* and not *formulas* (ie, 'The integral for arclength' is not a *theorem*).

11. Choose a concept from single-variable calculus that was revisited in multivariable calculus and explain the parallels and the differences between the concept in the two contexts. Write your answer to someone who is finishing Calculus II and is thinking about taking Calculus III.