Supplementary Exercises for Sections 13.4

1. Find $\frac{dy}{dx}$ in each case both by Implicit Differentiation and by the Implicit Function Theorem.
   (a) $x^3 + y^3 + 3xy = 6$
   (b) $\sin(xy) = 4$
   (c) $e^x + xy + \ln(y) = 12$

2. Describe the points on the unit circle, $x^2 + y^2 = 1$, where we *cannot* define one variable in terms of the other.

3. Describe the points on the unit sphere, $x^2 + y^2 + z^2 = 1$, where we *cannot* define one of the variables in terms of the other two.

4. Chemistry students will recognize the *ideal gas law*, given by

$$PV = nRT$$

   which relates the Pressure, Volume, and Temperature of n moles of gas. (R is the ideal gas constant). Thus, we can view pressure, volume, and temperature as variables, each one dependent on the other two.

   (a) If pressure of a gas is increasing at a rate of $0.2Pa/min$ and temperature is increasing at a rate of $1K/min$, how fast is the volume changing?
   (b) If the volume of a gas is decreasing at a rate of $0.3L/min$ and temperatuere is increasing at a rate of $.5K/min$, how fast is the pressure changing?
   (c) If the pressure of a gas is decreasing at a rate of $0.4Pa/min$ and the volume is increasing at a rate of $3L/min$, how fast is the temperature changing?

5. Verify the following identity in the case of the ideal gas law:

$$\frac{\partial P}{\partial V}\frac{\partial V}{\partial T}\frac{\partial T}{\partial P} = -1$$

6. The previous exercise was a special case of the following fact, which you are to verify here: If $F(x, y, z)$ is a function of 3 variables, and the relation $F(x, y, z) = 0$ defines each of the variables in terms of the other two, namely $x = f(y, z)$, $y = g(x, z)$ and $z = h(x, y)$, then

$$\frac{\partial x}{\partial y}\frac{\partial y}{\partial z}\frac{\partial z}{\partial x} = -1$$