Supplementary Exercises for Sections 13.5

1. Find the directions in which the directional derivative of $f(x,y) = x^2 + \sin(xy)$ at the point $(1,0)$ has the value of 1.

2. A bug is crawling on the surface of a hot plate, the temperature of which at the point x units to the right of the lower left corner and y units up from the lower left corner is given by

$$T(x,y) = 100 - x^2 - 3y^3.$$

   (a) If the bug is at the point $(2,1)$, in what direction should he move to cool off the fastest? How fast is he cooling off?

   (b) If the bug is at the point $(1,3)$, in what direction should he move in order to maintain his temperature?

3. Suppose that $g(x,y) = y - x^2$. Find the gradient at the point $(-1,3)$. Sketch the level curve to the graph of g when $g(x,y) = 2$, and plot both the tangent line and the gradient vector at the point $(-1,3)$. (Make your sketch large). What do you notice, geometrically?

4. Recall from class that the gradient $\nabla(f)$ is a vector valued function of two variables. Prove the following gradient rules. Assume $f(x,y)$ and $g(x,y)$ are differentiable functions.

   (a) $\nabla(fg) = f\nabla(g) + g\nabla(f)$

   (b) $\nabla(\frac{f}{g}) = \frac{g\nabla f - f\nabla g}{g^2}$

   (c) $\nabla((f(x,y))^n) = nf(x,y)^{n-1}\nabla f$