Supplementary Exercises for Sections 13.6

1. How many third-order derivatives does a function of 2 variables have? How many of these are distinct?

2. How many nth order derivatives does a function of 2 variables have? How many of these are distinct?

3. Let α and k be constants. Prove that the function

$$u(x, t) = e^{-\alpha^2 k^2 t} \sin(kx)$$

is a solution to the heat equation $u_t = \alpha^2 u_{xx}$

4. Let a be a constant. Prove that

$$u = \sin(x - at) + \ln(x + at)$$

is a solution to the wave equation $u_{tt} = a^2 u_{xx}$.

5. Let $f(x, y)$ be a continuous differentiable function. Analyze the level curves near a critical value if that critical value is a max or a min. What if the level curve is a saddle point?