Supplementary Exercises for Sections 13.7

1. Suppose that $f(x, y) = x^2 + y^2 + kxy$. Find and classify the critical points, and discuss how they change when k takes on different values.
2. Find the shortest distance from the point $(0, b)$ to the parabola $y = x^2$.
3. Find the shortest distance from the point $(0, 0, b)$ to the paraboloid $z = x^2 + y^2$.
4. Consider the function $f(x, y) = x^3 - 3x^2y$.
   (a) Show that $(0, 0)$ is the only critical point of f.
   (b) Show that the Discriminant test is inconclusive for f.
   (c) Determine the traces of f obtained by setting $x = k$ for various values of k.
   (d) What kind of critical point is $(0, 0)$?
5. Find the volume of the largest rectangular box with edges parallel to the axes that can be inscribed in the ellipsoid $2x^2 + 72y^2 + 18z^2 = 288$.