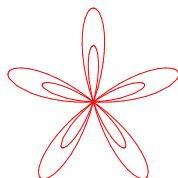Supplementary Exercises for Sections 14.2

1. Investigate and describe the differences between the graphs of $r = \cos(2\theta)$ and $r = \sin(2\theta)$.

2. Investigate and describe the differences between the graphs of $r = \cos(2k\theta)$ and $r = \cos((2k+1)\theta)$

3. Here is the plot of $r = 1 + 4\sin(5\theta)$.

   (a) Describe the behavior of the graph in terms of the given equation. Specifically, explain maximum and minimum values, number of leaves, and the 'leaves within leaves'.

   (b) Give an integral or integrals to determine the area outside a smaller leaf but inside a larger leaf.

   (c) How would changing the value of a in the equation $r = 1 + a\cos(5\theta)$ change the relative sizes of the inner and outer leaves? Focus on values $a \geq 1$. (Hint: How would we change the maximum and minimum values?)

4. Consider the integral
$$\iint_D \frac{1}{\sqrt{x^2 + y^2}}\, dA$$
where D is the unit disk centered at the origin.

   (a) Why might this integral be considered improper?

   (b) Calculate the value of this integral over the annulus with outer radius 1 and inner radius δ.

   (c) Obtain a value for the integral on the whole disk by letting δ approach 0.

   (d) For which values λ can we replace the denominator with $(x^2 + y^2)^\lambda$ in the original integral?