Supplementary Exercises for Sections 14.3

1. The plane $ax + by + cz = d$ cuts a triangle in the first octant provided that a, b, c and d are all positive. Set up the integral to find the area of this triangle.

2. The surface area formula can be used to compute the surface area of the upper half of the sphere $x^2 + y^2 + z^2 = a^2$, but an improper integral results as the function as we approach what would be the bounds.

   (a) Set up the appropriate integral to calculate this area and give two algebraic reasons why it is improper (one will look similar to our work from the Implicit Function Theorem).

   (b) Find the surface area of the upper hemisphere of $x^2 + y^2 + z^2 = a^2$ above a circle of radius t where $t < a$

   (c) Find the surface area of the whole upper hemisphere by taking a limit of your answer in part (b) as t approaches a.

3. For each of the integrals in Section 14.5, give a description of the volume (both algebraic and geometric) that is the domain of integration.

4. Find the region for which

$$\int\int\int_E (1 - x^2 - y^2 - z^2)\, dV$$

   is a maximum.