Math 225: Practice Exam 1
Spring 2012

These problems are culled from past Calc III exams and should give you a sense of the types of questions to expect on the upcoming midterm. This list is probably a bit longer than what you should expect on the exam itself. We will cover the solutions to these problems Thursday in class.

1. (a) Give the equation, in rectangular coordinates, of the cylinder of radius 6 centered around the z-axis.
   (b) Give the equation, in rectangular coordinates, of the cylinder of radius 6 centered around the y-axis.
   (c) Find, in parametric form, the equation of the curve of intersection of the cylinder in part (b) with the plane $y + 4z = 3$, and describe the curve.

2. Let ℓ_1 be the line through the two points (-3, 1, 0) and (1,1,2), and ℓ_2 be the line through the points (6, 2, 6) and (3, -1, 0).
   (a) Find the point of intersection of ℓ_1 and ℓ_2.
   (b) Find the plane that contains both lines.
   (c) Find the *acute* angle at which these two lines intersect.

3. Show that if vectors $\mathbf{x}-\mathbf{y}$ and $\mathbf{x}+\mathbf{y}$ are orthogonal, then $\mathbf{x}$ and $\mathbf{y}$ must have the same length.

4. (a) Find the equation of the tangent line to the curve $\mathbf{r}(t) = \langle 4 - t, 3t - t^2, t\rangle$ at the point when $t = 0$.
   (b) Using your work in part (a), find $\mathbf{T}(0)$. (Do NOT try to calculate a generic formula for $\mathbf{T}(t)$).
   (c) Find the plane normal to $\mathbf{r}(t)$ at $t = 0$.
   (d) Find the curvature and the tangential and normal components of acceleration at $t = 0$.

5. Suppose that a particle is moving with acceleration

$$\mathbf{a}(t) = \langle 6t, \cos(t), e^t\rangle$$

   and that the object starts with initial velocity vector $\langle 2, 1, 2\rangle$ and initial position vector $\langle 0, 1, 3\rangle$. Find the position of the object when $t = 1$.

6. Match the equation to the surface description. Warning: There is one outlier in each group!!

(a) $x^2 + 9y^2 + 81z^2 = 81$	I Hyperboloid of One Sheet
(b) $x^2 + 9y^2 + 81 = 81z^2$	II Hyperboloid of Two Sheets
(c) $x^2 + 9y^2 + 81 = 81z$	III Ellipsoid
(d) $x^2 - 9y^2 + 81 = 81$	IV Elliptical Paraboloid

7. Consider the functions $f(x,y) = x^2 + y^2$, $g(x,y) = \sqrt{x^2+y^2}$, and $h(x,y) = \ln(x^2+y^2)$.

   (a) What familiar surfaces are the graphs of f and g?

   (b) What familiar curve is a level curve for f, g, or h?

   (c) Plot the level curves for each of the three functions for $z = 1, 2, 3, 4$ and 5, with attention to spacing to show steepness.

   (d) Which function is steepest? Which is least steep? Explain.

8. (a) Draw the level curves to $f(x,y) = \frac{y}{x^2}$ for $f(x,y) = -2, -1, 0, 1, 2$.

   (b) What is happening with these level curves at the origin?