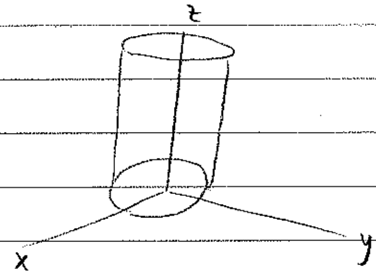
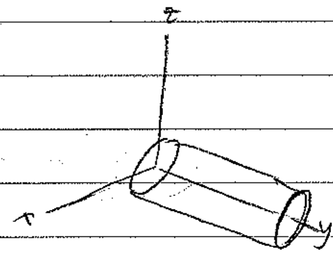
1a. $\boxed{x^2+y^2 = 36}$



b. $\boxed{x^2+z^2= 36}$



C. Intersect $y+4z=3$ with part b

$x = t$
$z = \pm\sqrt{36-t^2}$
$y = 3-4\pm\sqrt{36-t^2}$ $\longrightarrow$ $\boxed{\langle\ \underline{6\cos(t)}\ ,\ \underline{3-4\sin(t)}\ ,\ \underline{6\sin(t)}\ \rangle}$

another example)

$\langle \cos t, \sin t, 3\sin t\ \rangle$
$\rightarrow\ x^2+y^2=1$
$\rightarrow\ z = 3y$

2(a) ℓ_1 $(-3,1,0) \longleftrightarrow (1,1,2)$
ℓ_2 $(6,2,6) \longleftrightarrow (3,-1,0)$

intersection

x	$-3+4t$	$6+(-3)s$
y	$1+0t$	$2+(-3)s$
z	$0+2t$	$6+(-6)s$

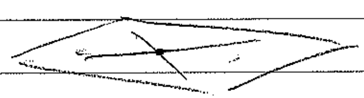
$\longrightarrow$ $1 = 2-3s$
$\boxed{s = 1/3}$

Plug back in to first equation

$-3+4t = 6-3(\tfrac{1}{3})$
$-3+4t = 6-1$
$-3+4t = 5$
$4t = 5+3$
$\boxed{t=2}$

at point $t=2$ and $s=1/3$, ℓ_1 and ℓ_2 intersect
at point $(5,1,4)$ $\longleftarrow$ got after plugging into x, y, and z

(b) plane for both lines



point $(5,1,4)$

$\vec{n}$ $\langle 4,0,2\rangle$
$\times \langle -3,-3,-6\rangle$
$\langle 6,18,-12\rangle$

$6(x-5)+18(y-1)-12(z+4)=0$

(c) $\langle 4,0,2\rangle \cdot \langle -3,-3,-6\rangle$ (looking for acute angle)
$= -12+0-12 = -24$
$\arccos\left(\frac{-24}{\sqrt{20}\sqrt{54}}\right)$

OR $\langle 4,0,2\rangle \cdot \langle 3,3,6\rangle = 24$
$\arccos\left(\frac{24}{\sqrt{20}\sqrt{54}}\right)$

3. $(\vec{x}-\vec{y}) \perp (\vec{x}+\vec{y})$

THEN $(\vec{x}-\vec{y})\cdot(\vec{x}+\vec{y}) = 0$

$\vec{x}\cdot\vec{x} + \cancel{\vec{x}\cdot\vec{y}} - \cancel{\vec{y}\cdot\vec{x}} - \vec{y}\cdot\vec{y} = 0$

$\vec{x}\cdot\vec{x} - \vec{y}\cdot\vec{y} = 0$

$\vec{x}\cdot\vec{x} = \vec{y}\cdot\vec{y}$

$|x|^2 = |y|^2$

$\boxed{|x| = |y|}$

remember:

Dot product:

angle between two vectors

$\vec{x}\cdot\vec{x} = |x|^2$

$\vec{x} \perp \vec{y} \Longleftrightarrow \vec{x}\cdot\vec{y} = 0$

$\vec{x}\cdot\vec{y} = \vec{y}\cdot\vec{x}$

Cross product

vector $\perp$ to two vectors

$\vec{x} \times \vec{x} = \vec{0}$

$\vec{x} \times \vec{y} = -(\vec{y} \times \vec{x})$

4.a. $\vec{r}(t) = \langle 4-t,\ 3t-t^2,\ t \rangle$

T line when $t=0$

Point: $\vec{r}(0) = \langle 4-0,\ 3(0)-(0)^2,\ 0 \rangle$

$= \boxed{\langle 4, 0, 0 \rangle}$

direction vector: $\vec{r}'(0) = \langle -1,\ 3-2t,\ 1 \rangle$

$= \boxed{\langle -1, 3, 1 \rangle}$

line $\vec{l}(t) = \langle 4, 0, 0 \rangle + t\langle -1, 3, 1 \rangle$

b. $\vec{T}(0) = \dfrac{r'(0)}{|r'(0)|}$

$= \boxed{\dfrac{1}{\sqrt{11}} \langle -1, 3, 1 \rangle}$

(smooth curve: $r'(t) \neq 0$)

c. Normal plane $\longrightarrow$ Tangent Line recast as a plane

point $(4, 0, 0)$

$\vec{n} = \langle -1, 3, 1 \rangle$

plane: $\boxed{-(x-4) + 3(y-3) + 1(z-1) = 0}$

d. $K = \dfrac{|r'(t) \times r''(t)|}{|r'(t)|^3}$

$a_T = \dfrac{r'(t) \cdot r''(t)}{|r'(t)|}$

$a_N = \dfrac{|r'(t) \times r''(t)|}{|r'(t)|}$

when $r = \langle 4-t, 3t-t^2, t \rangle$

$r' = \langle -1, 3-2t, 1 \rangle$

$r'' = \langle 0, 2, 0 \rangle$

$r' \times r'' = \langle 2, 0, 2 \rangle$

$r' \cdot r'' = -2(3-2t)$

$|r'| = \sqrt{11}$ when $t=0$

$\boxed{K = \dfrac{\sqrt{8}}{(\sqrt{11})^3}}$

$\boxed{a_T = \dfrac{-6}{\sqrt{11}}}$

$\boxed{a_N = \dfrac{\sqrt{8}}{\sqrt{11}}}$

5. $\vec{a}(t) = \langle 6t, \cos(t), e^t \rangle$

$\vec{V}(t) = \int_0^t \vec{a}(x)\,dx + \vec{V}_0$

$= \int_0^t \langle 6x, \cos x, e^x \rangle dx + \cancel{\langle 2, 1, 2 \rangle}$

$3x^2, \sin x, e^x \Big|_0^t + \langle 2, 1, 2 \rangle$

$\boxed{\vec{V}(t) = \langle 3t^2+2, \sin t + 1, e^t + 1 \rangle}$

$\vec{S}(t) = \int_0^t \vec{V}(x) + S_0 = \int_0^t \langle 3x^2+2, \sin x + 1, e^x + 1 \rangle + \langle 0, 1, 3 \rangle$

$= \langle x^3 + 2x, -\cos x + x, e^x + x \rangle \Big|_0^t + \langle 0, 1, 3 \rangle$

$\boxed{\vec{S}(t) = \langle t^3 + 2x, -\cos t + t + 2, e^t + t + 2 \rangle}$

$\boxed{\vec{S}(1) = \langle 3, -\cos(1) + 3, e + 3 \rangle}$

6. a. $x^2 + 9y^2 + 81z^2 = 81$ — III Ellipsoid

b. $x^2 + 9y^2 + 81 = 81z^2$ — II Hyperboloid of Two Sheets

c. $x^2 + 9y^2 + 81 = 81z$ — IV Elliptical Paraboloid

d. $x^2 - 9y^2 + 81 = 81$

$x^2 - 9y^2 = 0$

$x^2 = 9y^2$

$x = \pm 3y$

7. $f(x,y) = x^2+y^2$

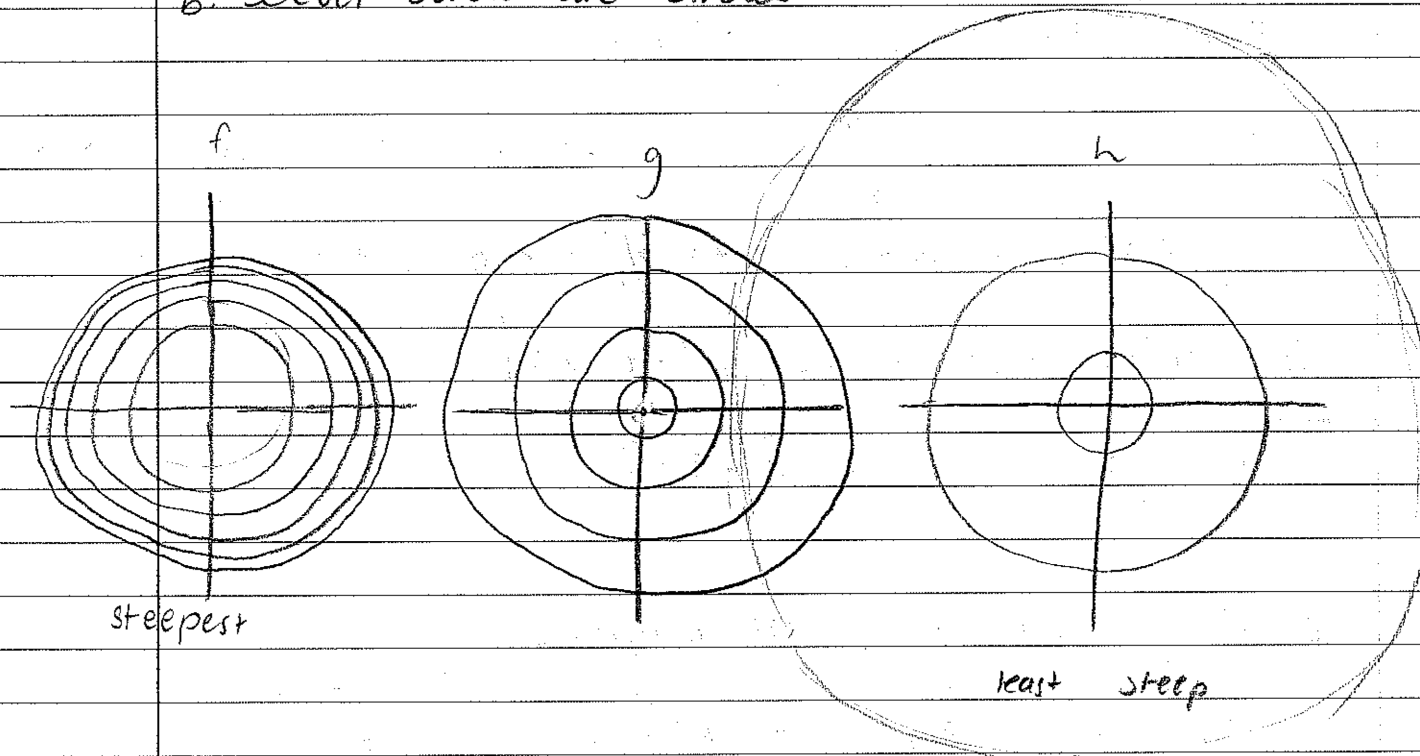
$g(x,y) = \sqrt{x^2+y^2}$

$h(x,y) = \ln(x^2+y^2)$

a. f is a paraboloid

g is a cone

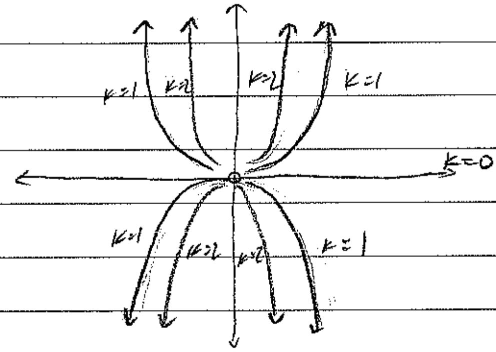
b. level curves are circles



8a. $f(x,y) = y/x^2$

$k = y/x^2$

$kx^2 = y$



b. There is an open hole at the origin because the function is not defined at $x=0$. Can't divide by zero.

Even Practice Exam Key Calculus III Balof

#2 a) l_1 $(-3,1,0) \leftrightarrow (1,1,2)$

l_2 $(6,2,6) \leftrightarrow (3,-1,0)$

Intersection

$-3 + 4t \neq 6 + (-3)s$

$1 + 0t \neq 2 + (-3)s$ $\quad 1 = 2 - 3s$, $s = 1/3$ $\quad$ point

$0 + 2t \neq 6 + (-6)s$ $\quad$ plug back in $\quad (5, 1, 4)$

$-3 + 4t = 5$; $t = 2$

b) find plane w/ both lines (need point & normal vector)

point: $(5,1,4)$

$\vec{n}$: $\langle 4,0,2 \rangle \times \langle -3,-3,-6 \rangle = \langle 6, 18, -12 \rangle$

normal to plane

thus:

$$6(x-5) + 18(y-1) - 12(z-4) = 0$$

c) acute angle of intersection?

$\langle 4,0,2 \rangle \cdot \langle -3,-3,-6 \rangle = -24$ $\quad$ obtuse!

two methods

① $\pi - \dfrac{\arccos(-24)}{\sqrt{20}\sqrt{54}}$ $\quad$ or $\quad$ ② use different dot product:

$\langle 4,0,2 \rangle \cdot \langle 3,3,6 \rangle = 24$

$= \arccos\left(\dfrac{24}{\sqrt{20}\sqrt{54}}\right)$

#4 ⓐ tangent line to $\vec{r}(t) = \langle 4-t, 3t-t^2, t \rangle$ when $t = 0$

need point & direction vector

$\vec{r}(0) = \langle 4-0, 3(0)-0^2, 0 \rangle = \langle 4,0,0 \rangle$

$\vec{r}'(0) = \langle -1, 3-2t, 1 \rangle \big|_{t=0}$

direction vector $= \langle -1, 3, 1 \rangle$

line $l(t) = \langle 4,0,0 \rangle + t\langle -1,3,1 \rangle$

b) $\vec{T}(0)$? $= \frac{r'(0)}{|r'(0)|} = \frac{1}{\sqrt{11}}\langle -1, 3, 1\rangle$ (9)

(smooth curve)

c) normal plane : need a point & direction $\vec{v}$
(its tangent line recast as a plane)

point : (4, 0, 0)

$\vec{n} = \langle -1, 3, 1\rangle$ plane : $-(x-4) + 3(y-0) + 1(z-0) = 0$

d) $K = \frac{|r' \times r''|}{|r'|^3}$ at $t = 0$

$a_T = \frac{r' \cdot r''}{|r'|}$ $a_N = \frac{|r' \times r''|}{|r'|}$

$r(t) = \langle 4-t, 3t - t^2, t\rangle$

$r'(t) = \langle -1, 3-2t, 1\rangle$

$r''(t) = \langle 0, -2, 0\rangle$

$r' \times r'' = \langle 2, 0, 2\rangle$ $|r'| = \sqrt{11}$

$r' \cdot r'' = -2(3-2t)$

$K = \frac{\sqrt{8}}{\sqrt{11}^3}$ $a_T = \frac{-6}{\sqrt{11}}$ $a_N = \frac{\sqrt{8}}{\sqrt{11}}$

#6) a) $x^2 + 9y^2 + 81z^2 = 81$ Ellipsoid

b) $x^2 + 9y^2 + 81 = 81z^2$ Hyperboloid of two sheets

c) $x^2 + 9y^2 + 81 = 81z$ Elliptical paraboloid

d) $x^2 - 9y + 81 = 81$

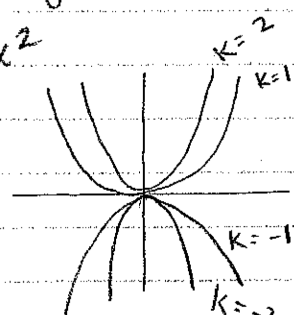
$= x^2 - 9y = 0$

$x = \pm 3y$ ← degenerate hyperbola

#8) $f(x,y) = y/x^2$

$K = y/x^2$

$y = kx^2$



f(x,y) is undefined at (0,0)