

**Math 225: Exam the Second**  
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This exam is closed book and closed notes. READ ALL DIRECTIONS CAREFULLY!! Please justify all of your answers. You may use a calculator for arithmetic and scientific functions only (ie, no graphing). You have two hours.

1. Let  $f(x, y) = \sqrt{9 - x^2 - y^2}$ 
  - (a) What is the domain of  $f$ ?
  - (b) For which  $k$  can we draw level curves of the form  $f(x, y) = k$ ?
  - (c) Draw level curves below for  $k = 0, 1, 2, 3$ , with attention to spacing.
2. Find the mixed partial derivative  $f_{xy}$  if  $f(x, y) = x^2 \arccos(x) + xy + y^y$ .
3. Let  $f(x, y) = e^{xy}$ .
  - (a) Find the tangent plane at the point  $(2, 1)$ , and use it to approximate  $f(2.1, 1.04)$ . (Please express your answer as a multiple of  $e^2$ .)
  - (b) Find  $D_u(f)$  as we move from  $(2, 1)$  to  $(5, 5)$ .
4. Let  $x^2yz + yz + x^3y = 5$ .
  - (a) Find  $\frac{\partial y}{\partial x}$  and  $\frac{\partial y}{\partial z}$ .
  - (b) Find the tangent plane to  $f(x, y)$  at  $(-1, 1, 3)$
5. Let  $f(x, y) = x^2 + kxy + y^2$ , where  $k$  is a constant.
  - (a) Find the critical point of  $f$  (it should be independent of  $k$ ).
  - (b) For which values of  $k$  is the critical point a minimum?
  - (c) For which values of  $k$  is the critical point a maximum?
  - (d) For which values of  $k$  is the critical point a saddle point?
  - (e) For which values of  $k$  does the discriminant test fail to be conclusive at the critical point?
  - (f) What is the nature of the critical point for those values of  $k$ ?
6. Find the maximum and minimum values of  $f(x, y) = 2x + y$  subject to the constraint  $x^2 + 2y^2 = 1$ . You may use any method that you wish.

7. Find

$$\iint_R x(xy + 1)^3 dA$$

where  $R = [0, 1] \times [0, 2]$ .

8. Consider the region  $R$  bound by the lines  $y = 0$ ,  $x = 9$  and  $x = y^2$ . Write

$$\iint_R f(x, y) dA$$

in both possible orders. (That is, use  $R$  to determine the bounds of integration).

9. Let  $z^2 = 1 + x^2 + y^2$ .

(a) What quadric surface is the graph of this equation?

(b) Find the volume in the first octant bound by this surface and the plane  $z = 3$ .

10. Convert

$$\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} x + y \, dy \, dx$$

to polar coordinates, and solve the integral.

11. Set up the integral to find the area of the surface of  $f(x, y) = x^2 + 2y^2$  over the triangle with vertices  $(0, 0)$ ,  $(1, 1)$  and  $(1, -1)$ . You need not compute this integral.

12. Should you get the same integral for the area of the surface of  $f(x, y) = x^2 + 2y^2 + 3$ ? Explain both geometrically and algebraically.