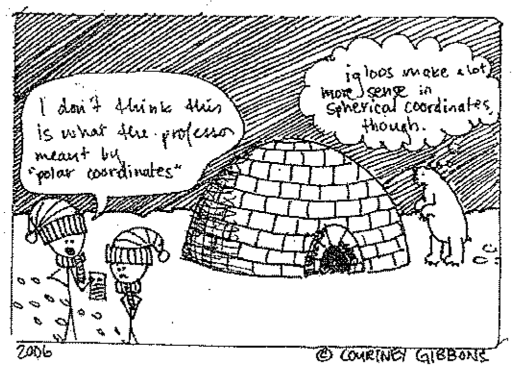
KEY

Math 225: Quiz the Penultimate

April 26, 2012

This quiz is closed book and closed notes. Please justify all of your answers. You have the remainder of the period.



1. Find

$$\iiint_E e^{(x^2+y^2+z^2)^{\frac{3}{2}}}\,dV$$

where E is the region bound by the unit sphere in the first octant.

Sphericals

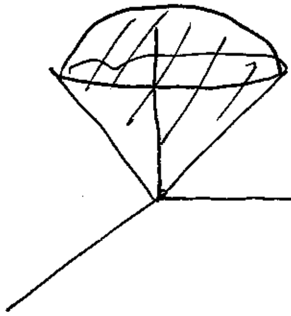
$$\iiint_E e^{\rho^3}\underline{\rho^2 \sin\phi}\,d\rho\,d\theta\,d\phi$$

$$= \int_0^{\pi/2}\int_0^{\pi/2}\int_0^1 e^{\rho^3}\rho^2 \sin\phi\,d\rho\,d\theta\,d\phi$$

$$= \int_0^{\pi/2}\int_0^{\pi/2} \frac{e-1}{3}\sin\phi\,d\theta\,d\phi$$

$$= \int_0^{\pi/2} \frac{\pi(e-1)}{6}\sin\phi\,d\phi = \frac{\pi(e-1)}{6}$$

2. Sketch and find the volume bound $x^2 + y^2 + z^2 = 9$ and $z^2 = x^2 + y^2$ using a triple integral in spherical coordinates.



$0 \le \rho \le 3$

$0 \le \theta \le 2\pi$

$0 \le \phi \le \pi/4$

$$\int_0^{\pi/4}\int_0^{2\pi}\int_0^{3} 1\,\rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

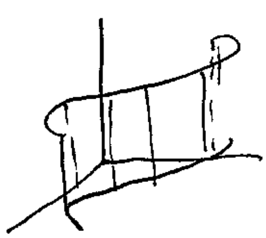
$$= \int_0^{\pi/4}\int_0^{2\pi} 9 \sin\phi \, d\theta \, d\phi$$

$$= 18\pi \int_0^{\pi/4} \sin\theta \, d\theta = 18\pi \left[-\cos\phi\right]_0^{\pi/4}$$

$$= 18\pi\left[\frac{\sqrt{2}}{2} - 1\right]$$

$$= \underline{18\pi - 9\sqrt{2}\,\pi}$$

3. Our "World's Most Boring Example" has had several geometric interpretations thus far. Namely, $\iint_R 1\; dA$ gives the area of R, and $\iiint_E 1\; dV$ gives the volume of E. What, geometrically, $\int_C 1\; ds$?



$\int_C 1\, ds$ represents the length of the curve C

4

$$\int_a^b \sqrt{(x')^2 + (y')^2}\, dt$$

4. Find each of the following line integrals.

(a) $\int_C xy\, ds$, where C is the line segment from $(1,2)$ to $(4,3)$.

$x = 1+3t \qquad 0 \le t \le 1$

$y = 2+t$

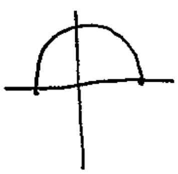
$ds = \sqrt{3^2+1^2}\,dt = \sqrt{10}\,dt$

$$\int_0^1 (1+3t)(2+t)\sqrt{10}\,dt$$

$$= \sqrt{10}\int_0^1 2+7t+3t^2\,dt$$

$$= \sqrt{10}\left[2t+\frac{7}{2}t^2+t^3\right]_0^1 = \frac{13}{2}\sqrt{10}$$

(b) $\int_C y\, dx\ + 2x\, dy$ where C is the top half of the unit circle, traversed counterclockwise.



$x = \cos t$

$y = \sin t$

$0 \le t \le \frac{\pi}{2}$

$$\int_C y\,dx + 2x\,dy$$

$$= \int_0^{\pi/2} \sin t(-\sin t)\,dt + 2\cos t(\cos t)\,dt$$

$$= \int_0^{\pi/2} -\sin^2 t + 2\cos^2 t\,dt$$

$$= \int_0^{\pi/2}\left(\frac{1-\cos 2t}{2}\right) + (1+\cos 2t)\,dt = \left[\frac{t}{2} - \frac{\sin 2t}{4}\right] + \left[t + \frac{\sin 2t}{2}\right]_0^{\pi/2}$$

$$= 3\pi/4$$

(c) $\int_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F} = \langle 2x, y^2\rangle$ and C is the parabolic segment of $y = x^2$ between (1,1) and (4, 16)

$x = t$

$y = t^2$

$1 \le t \le 4$

$$\int_1^4 \langle 2t, t^4\rangle \cdot \langle 1, 2t\rangle\,dt$$

$$= \int_1^4 2t + 2t^5\,dt$$

$$= t^2 + \frac{t^6}{3}\Big]_1^4\,dt \qquad 16 + \frac{4096}{3} - 1 - \frac{1}{3}$$

$$= 15 + \frac{4095}{3} = 15 + 1365 = \underline{1380}$$

5. (Bonus) You may have 0.5, 1.0, or 1.5 points extra credit. Those choosing the option with the lowest positive number of votes will receive the extra credit.