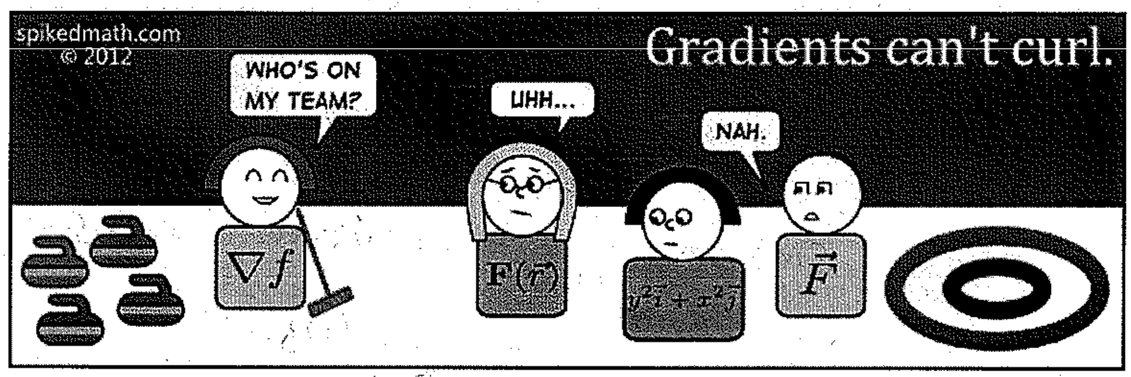
KEY

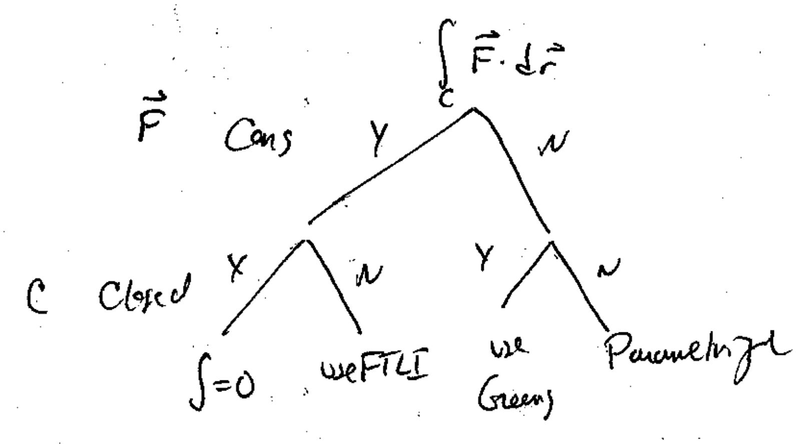
Math 225: Quiz the Last
May 3, 2012

This quiz is closed book and closed notes. Please justify all of your answers. You have the remainder of the period.



1. What two questions do we ask when determining which method to use to solve a line integral of the form $\int_C \mathbf{F} \cdot d\mathbf{r}$?

1. Is the field $\vec{F}$ conservative?
2. Is the curve, C, closed



2. Solve each of the following line integrals. Be specific about which method you're using in each case.

(a)

$$\int_C \overset{P}{\cos(x)\sin(y)}\, dx + \overset{Q}{\sin(x)\cos(y)}\, dy$$

where C is the line segment from (0,1) to (1,2).

$\frac{\partial Q}{\partial x} = \cos x \cos y \qquad \frac{\partial P}{\partial y} = \cos x \cos y$ ✓

use FTLI

$\frac{\partial f}{\partial x} = \cos x \sin y$; $f = \sin x \sin y + g(y)$ $\quad \frac{\partial f}{\partial y} = \sin x \cos y$

$f = \sin x \sin y \Big|_{(0,1)}^{(1,2)} = \sin(1)\sin(2) - \underbrace{\cancel{\sin(0)}}_{0}\sin(1)$

$= \boxed{\sin(1)\sin(2)}$

(b)

$$\oint_C \overset{P}{(e^x + 2y)}\, dx + \overset{Q}{(e^{3y} + 3x)}\, dy$$

where C is the triangle from (0,0) to (1,0) to (1,1) back to (0,0)

use Green's Thm

$$\iint_R \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\, dA = \int_0^1 \int_0^x 3 - 2\, dy\, dx$$

$$= \int_0^1 \int_0^x 1\, dy\, dx = \boxed{\frac{1}{2}}$$

(c)

$$\oint_C \overset{P}{x^4 \cos(x)}\, dx + \overset{Q}{y \tan(y)}\, dy$$

where C is the circle of radius 2, centered at the origin, traversed counterclockwise.

$\frac{\partial Q}{\partial x} = 0 = \frac{\partial P}{\partial y}$, C is closed

$\to \int_C P\,dx + Q\,dy = \boxed{0}$

(d)

$$\int_C yz\,dx + xz\,dy + xy\,dz$$

where C is parametrized by $\langle t, t^2, t^3 \rangle$ from $t = 1$ to $t = 2$.

FTLI: $f = xyz \Big|_{(1,1,1)}^{(2,4,8)} = 64 - 1 = \boxed{63}$

ALT: $\int_1^2 t^5 \cdot 1 + t^4 \cdot 2t + t^3 \cdot 3t^2 \, dt$

$= \int_1^2 6t^5 \, dt = t^6 \Big|_1^2 = 64 - 61 = \boxed{63}$

P Q R

3. Let $\mathbf{F} = \langle x^2 + y^2, y^2 + z^2, x^2 + z^2 \rangle$. Calculate **curl**(**F**) and div(F). Also, verify that div(**curl**((**F**)) = 0.

curl $\vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2+y^2 & y^2+z^2 & x^2+z^2 \end{vmatrix} = \langle 0-2z, 0-2x, 0-2y \rangle = \langle -2z, -2x, -2y \rangle$

div $\vec{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = 2x + 2y + 2z$

div curl $F = \underbrace{\frac{\partial}{\partial x}(-2z)}_{0} + \underbrace{\frac{\partial}{\partial y}(-2x)}_{0} + \underbrace{\frac{\partial}{\partial z}(-2y)}_{0} = 0$

4. (Bonus) To check if a vector field on $\mathbf{R}^2$ is conservative, we need to check the equality of 1 pair of mixed partials. To check if a vector field on $\mathbf{R}^3$ is conservative, we need to check 3 pairs of mixed partials. What about for a vector field on $\mathbf{R}^4$? On $\mathbf{R}^n$?

$\mathbb{R}^4 \to$ 6 pairs of mixed partials

$\mathbb{R}^n \to \frac{n(n-1)}{2}$ pairs of mixed partials.