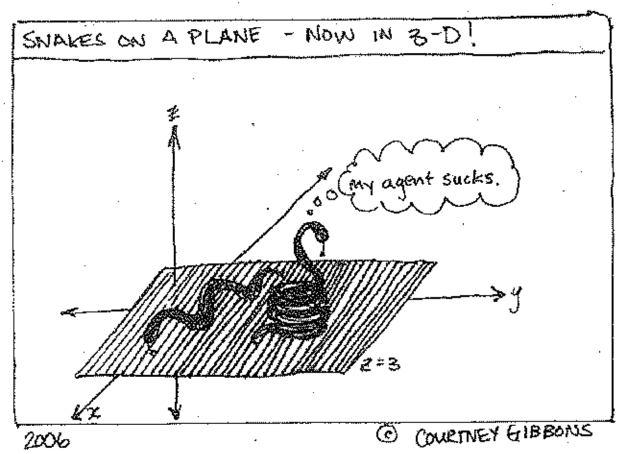
KEY

Math 225: Quiz the First January 26, 2012

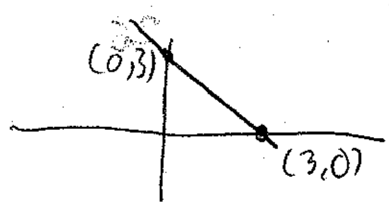
This quiz is closed book and closed notes. You may use your calculator for the purposes of arithmetic operations (including trig). When asked for specific values, however, you must show the relevant algebra. **READ ALL QUESTIONS CAREFULLY!** You have the remainder of the period.



1. Plot and describe the set of points satisfied by $x + y = 3$

   (a) in $\mathbb{R}^2$.

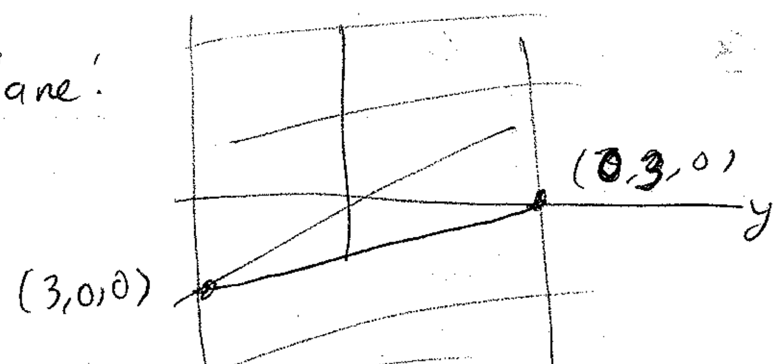
   Line!



   (b) in $\mathbb{R}^3$.

(4)

   Plane!



2. Find a unit vector in the same direction as $\langle 2, 3, -2\rangle$

(2)

$\vec{v} = \langle 2, 3, -2 \rangle \qquad |\vec{v}| = \sqrt{4+9+4} = \sqrt{17}$

$\vec{u} = \frac{1}{\sqrt{17}} \langle 2, 3, -2 \rangle$

3. Give the equation for the sphere centered at $(4, 6, -1)$ with radius 5. Also, tell whether this sphere intersects each of the coordinate (xy, xz, and yz) planes.

Eq'n $(x-4)^2+(y-6)^2+(z+1)^2 = 25$

3

This sphere intersects the yz and the xy planes, but not the xz plane (the radius is too small)

4. Let $A = (1, 0, 1)$, $B = (2, -1, 3)$, and $C = (4, 1, 5)$

(a) Find $\overrightarrow{AB}$, $\overrightarrow{AC}$ and their magnitudes.

$\overrightarrow{AB} = \langle 1, -1, 2 \rangle$ $\quad |\overrightarrow{AB}| = \sqrt{6}$

$\overrightarrow{AC} = \langle 3, 1, 4 \rangle$ $\quad |\overrightarrow{AC}| = \sqrt{26}$

(b) Find $\overrightarrow{AB} \cdot \overrightarrow{AC}$.

5

$\overrightarrow{AB} \cdot \overrightarrow{AC} = (3 - 1 + 8) = 10$

(c) Without calculating the angle, is the angle formed at A acute, right, or obtuse? Explain.

$\measuredangle A$ is acute since $\overrightarrow{AB} \cdot \overrightarrow{AC} > 0$

5. Let $\mathbf{x} = \langle 2, 3, 4\rangle$ and let $\mathbf{y} = \langle 1, -1, 1\rangle$

(a) Find $\text{proj}_{\mathbf{y}}\mathbf{x}$ and $\text{comp}_{\mathbf{y}}\mathbf{x}$.

$$\text{proj}_{\vec{y}}\vec{x} = \frac{\vec{x}\circ\vec{y}}{|\vec{y}|^2}\vec{y} = \frac{2-3+4}{3}\langle 1,-1,1\rangle$$

$$= \frac{3}{3}\langle 1,-1,1\rangle = \langle 1,-1,1\rangle$$

(6) $\text{Comp}_{\vec{y}}\vec{x} = |\text{proj}_{\vec{y}}\vec{x}| = \sqrt{3}$.

NB: $\langle 2,3,4\rangle - \langle 1,-1,1\rangle = \langle 1,-4,3\rangle \perp \langle 1,-1,1\rangle$

(b) Find a non-zero vector perpendicular to both $\mathbf{x}$ and $\mathbf{y}$.

$$\vec{x}\times\vec{y} = \begin{array}{c}\langle 2,3,4\rangle \\ \times\langle 1,-1,1\rangle \\ \hline (3- {}^{-}4,\ 4-2,\ -2-3)\end{array}$$

$$= \langle 7,\ 2,\ -5\rangle$$

Check: $\langle 7,2,-5\rangle \cdot \langle 2,3,4\rangle = 14+6-20=0$ ✓

$\cdot\langle 1,-1,1\rangle = 7-2-5=0$

6. Prove that if $\mathbf{x}+\mathbf{y}$ is orthogonal to $\mathbf{x}-\mathbf{y}$, then $|\mathbf{x}| = |\mathbf{y}|$ (Hint: Calculate an appropriate dot product).

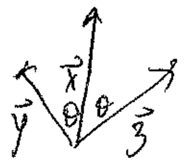
(5) $$\langle\vec{x}+\vec{y}\rangle\circ\langle\vec{x}-\vec{y}\rangle = \vec{x}\cdot\vec{x} + \cancel{\vec{x}\cdot\vec{y}} \cancel{-\vec{y}\cdot\vec{x}} - \vec{y}\cdot\vec{y} \overset{\text{given}}{=} 0$$

$$|\vec{x}|^2 - |\vec{y}|^2 = 0 \Rightarrow |\vec{x}|^2 = |\vec{y}|^2.$$

7. (Bonus)

(a) If $(\mathbf{x} \cdot \mathbf{y}) = (\mathbf{x} \cdot \mathbf{z})$, must it be the case that $\mathbf{y} = \mathbf{z}$?

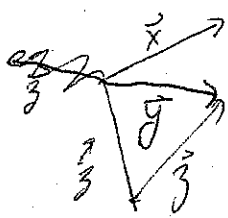
no.



Same dot product.

(b) If $(\mathbf{x} \times \mathbf{y}) = (\mathbf{x} \times \mathbf{z})$, must $\mathbf{y} = \mathbf{z}$?

no



same crossproduct.

if $\vec{y} - \vec{z}$ is parallel to $\vec{x}$.

(c) If both $(\mathbf{x} \cdot \mathbf{y}) = (\mathbf{x} \cdot \mathbf{z})$ and $(\mathbf{x} \times \mathbf{y}) = (\mathbf{x} \times \mathbf{z})$, must $\mathbf{y} = \mathbf{z}$?

Yes. Here we can show that $\vec{y} - \vec{z}$ is orthogonal to $\vec{x}$ and parallel to $\vec{x}$.

Thus $\vec{y} - \vec{z} = \vec{0}$.