KEY

Math 225: Quiz the Second

February 2, 2012

This quiz is closed book and closed notes. You may use your calculator for the purposes of arithmetic and for plotting equations, if helpful. When asked for specific values, however, you must show the relevant algebra. PLEASE READ ALL QUESTIONS CAREFULLY. You have 40 minutes.

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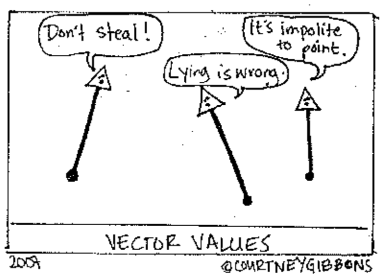


1. Let $\mathbf{x}, \mathbf{y}$ and $\mathbf{z}$ be vectors in $\mathbb{R}^3$. For each quantity listed, tell whether it is a vector, a scalar, or if the quantity does not make sense.

   (a) $(\mathbf{x} - \mathbf{y}) \times \mathbf{z}$

   $\vec{v} \times \vec{v} =$ vector

   (b) $(\mathbf{x} \cdot \mathbf{y}) + \mathbf{z}$

   $s + \vec{v}$ Nonsense

   (c) $\frac{1}{|\mathbf{x}|}(\mathbf{y} \cdot \mathbf{z})$

   $\frac{1}{s}(s) =$ scalar

   (d) $(\mathbf{x} \times \mathbf{z}) \times (\mathbf{y} \times \mathbf{z})$

   $(\vec{v}) \times (\vec{v}) =$ vector

   (e) $\mathbf{x}^2$

   nonsense! we can't "square" a vector.

2. Find a unit vector perpendicular to $\langle 1, -3, 2\rangle$ and $\langle -2, 4, -3\rangle$.

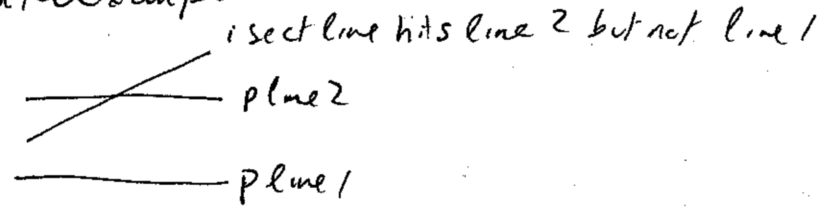
$$\begin{array}{r} \langle 1, -3, 2 \rangle \\ \times \langle -2, 4, -3 \rangle \\ \hline \end{array}$$

$$\langle 9-8, -4+3, 4-6 \rangle$$

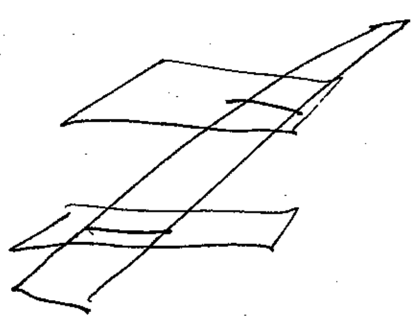
$$= \langle 1, -1, -2 \rangle \xrightarrow{\vec{u}} \frac{1}{\sqrt{6}} \langle 1, -1, -2 \rangle$$

3. a) Suppose that we have two parallel lines, and that a third line intersects one of the two. Must it also intersect the other? Why or why not? b) What if we replace 'lines' by 'planes'? (Pictures will help with this explanation!)

a no. Counterexample



b) yes, here we have parallel planes. Two planes intersect in a line or not at all. The lines of intersection will be parallel



Since planes extend infinitely in two directions, intersection with one plane means intersection with any parallel to it.

4. Find the plane perpendicular to the line $\ell(t) = \langle 1, -1, 3\rangle + t\langle 2, -1, 4\rangle$ and containing the point (2,2,-1).

$$\vec{n} = \langle 2, -1, 4\rangle$$

$$p = (2, 2, -1)$$

plane : $+2(x-2) - (y-2) + 4(z-1) = 0$

5. Find the line parallel to $\frac{x-1}{2} = 1 - y = \frac{z-3}{4}$ through the point (1, 2, 3).

$$\vec{v} = \langle 2, -1, 4\rangle$$

Point: $(1, 2, 3)$

line: $x = 1 + 2t$
$y = 2 - t$
$z = 3 + 4t$

6. (a) Find the line of intersection of the two planes

$$x + y + z = 3; x + 2y + 4z = 7$$

(Hint: For ease, use x=1 for your 'convenient' point on this line)

line: direction vector, given by $\vec{n}_1 \times \vec{n}_2$

$\vec{n}_1 = \langle 1,1,1 \rangle$

$\vec{n}_2 = \langle 1,2,4 \rangle$

$\langle 2,-3,1 \rangle$

line: $\langle 1,1,1 \rangle + t\langle 2,-3,1 \rangle = \ell(t)$

point ($x = 1 \Rightarrow$ $y + z = 2$, $2y + 4z = 6$ $\quad 2y = \cancel{2}$, $y = 1$, $z = 1$ $\quad (1,1,1)$

(b) Does your line in part (a) intersect the line

$$\langle x, y, z \rangle = \langle 1, 0, 3 \rangle + t\langle -2, 2, 1 \rangle$$

and, if so, where?

To tell intersection

line 1 (t)	line 2 (s)
$1 - 2t$	$= 1 + 2s$
$2t$	$= 1 - 3s$
$3 + t$	$= 1 + s$

$\hookrightarrow s = t + 2$

$1 - 2t = \cancel{1} + 2(t+2) \Rightarrow t = -1 \quad s = 1$

$\boxed{\text{point: } (3,-2,2)}$

(c) (Bonus) Given two skew lines, how do you find the plane parallel to both of them?

NB: there is more than one!

Two skew lines have nonparallel direction vectors $\vec{v}_1$ and $\vec{v}_2$, which will both be in the plane.

Thus $\vec{n} = \vec{v}_1 \times \vec{v}_2$. Take any point in space, this will give you a plane // to both.