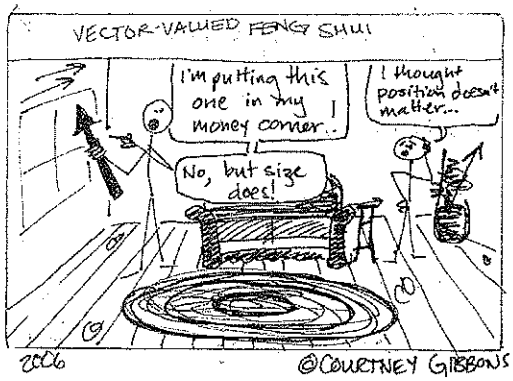
KEY

Math 225: Quiz the Third

This quiz is closed book and closed notes. You may use your calculator for the purposes of arithmetic and for plotting equations, if helpful. When asked for specific values, however, you must show the relevant algebra. READ ALL DIRECTIONS CAREFULLY. You have the remainder of the period.



1. Consider the curve $\mathbf{r}(t) = \langle t, t^2, 2t+1 \rangle$.

   (a) Set up, but don't compute, the integral for the length of this curve from $t = 1$ to $t = 3$.

$$\int_1^3 |\vec{r}\,'(t)|\,dt = \int_1^3 \sqrt{1^2+(2t)^2+(2)^2}\,dt$$

$$= \int_1^3 \sqrt{5+4t^2}\,dt.$$

   (b) What (if any) are the differences between this curve and $\mathbf{r}(t) = \langle t^2, t^4, 2t^2+1 \rangle$?

The first curve takes on all values for x, z, and only ~~positive~~ nonnegative values for y.

The second curve takes on only nonnegative values for each of x, y, z.

2. The trajectory of an object is given by the vector valued function

$$\mathrm{r}(t) = \langle \ln(t), \sqrt{1-t^2}, t^2 - 2 \rangle$$

(a) For which values of t is the curve defined?

We need $t > 0$ for the curve to be defined in x.

We need $|t| \leq 1$ for the curve to be defined in y.

We have no restrictions on z.

Thus $0 < t \leq 1$

(b) What values do x, y and z take on?

in this range $-\infty < x \leq 0$ ($\ln x < 0$ if $x < 1$)

$0 \leq y < 1$

$-2 \leq z \leq -1$

(c) Find the speed of the object when $t = \frac{1}{2}$

$$|\vec{r}\,'(t)| = \left| \left\langle \frac{1}{t}, \frac{1}{2}(1-t^2)^{-1/2}(-2t), 2t \right\rangle \right|_{t=1/2}$$

$$= \left| \left\langle 2, \frac{-\frac{1}{2}}{\sqrt{1-\frac{1}{4}}}, 1 \right\rangle \right| \quad \frac{-1/2}{\sqrt{3}/2} = -1/\sqrt{3}$$

$$= \sqrt{4 + \frac{1}{3} + 1} = \sqrt{16/3} = 4/\sqrt{3}.$$

(d) Find the equation of the tangent line to the object when $t = \frac{1}{2}$. (Try to leave the value for $\mathrm{r}(\frac{1}{2})$ in exact form).

$$\vec{r}\left(\tfrac{1}{2}\right) = \left\langle \ln \tfrac{1}{2}, \tfrac{\sqrt{3}}{2}, -\tfrac{7}{4} \right\rangle$$

$$\vec{r}\,'\left(\tfrac{1}{2}\right) = \langle 2, -1/\sqrt{3}, 1 \rangle$$

from above

line: $x = \ln\frac{1}{2} + 2t$

$y = \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{3}}t$

$z = -7/4 + t$.

3. Give a parametrization for the curve of intersection of $y^2 + z^2 = 1$ and $x + 2y + z = 3$.

Alg relationship indicates

set $y = \cos t$

$z = \sin t$

$(y^2 + z^2 = 1)$

$x = 3 - 2y - z$

$= 3 - 2\cos t - \sin t$

Curve: $\vec{r}(t) = \langle 3 - 2\cos t - \sin t, \cos t, \sin t \rangle$

4. Suppose that the velocity of an object is given by $v(t) = \langle \cos(2t), e^t, 2t+1 \rangle$. Find the position of the object as a function of time if the object is initially at $\langle 0, 2, 3 \rangle$.

$\vec{v}(t) = \langle \cos(2t), e^t, 2t+1 \rangle$

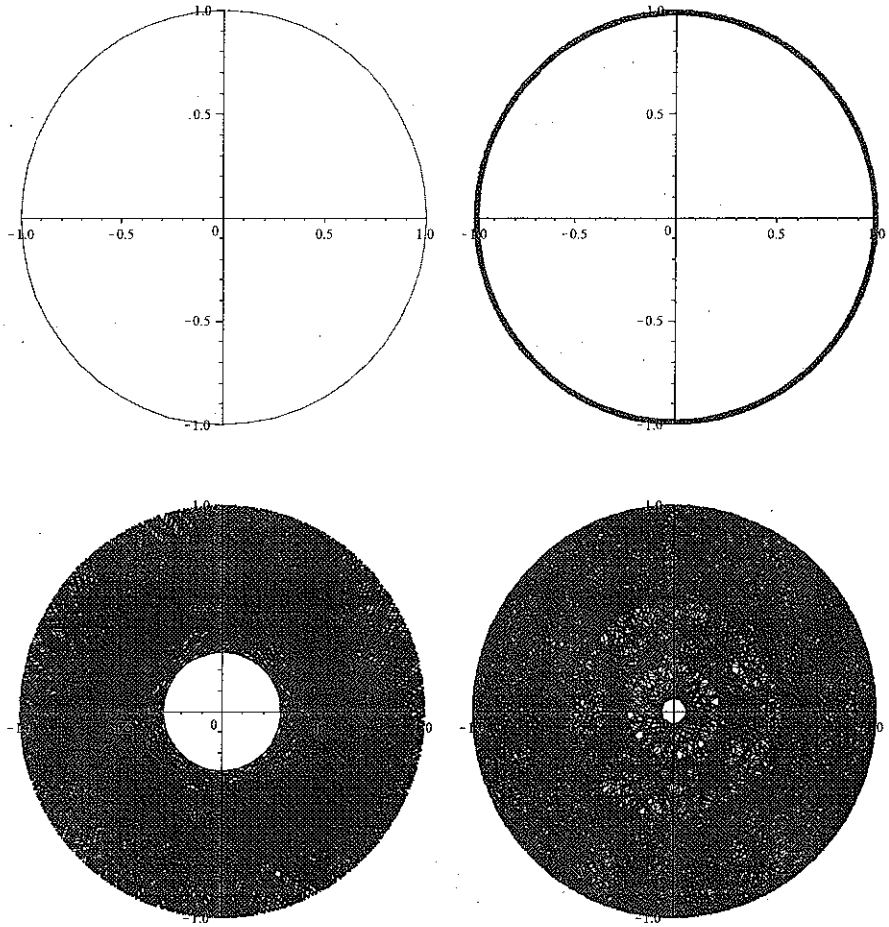
$$\vec{r}(t) = \int_0^t \langle \cos 2x, e^x, 2x+1 \rangle \, dx + \langle 0, 2, 3 \rangle$$

$$\rightarrow \frac{\sin 2x}{2}, e^x, x^2 + x \Big|_0^t + \langle 0, 2, 3 \rangle$$

$$= \left\langle \frac{\sin 2t}{2}, e^t, t^2 + t \right\rangle - \langle 0, 1, 0 \rangle + \langle 0, 2, 3 \rangle$$

$$= \left\langle \frac{\sin 2t}{2}, e^t + 1, t^2 + t + 3 \right\rangle$$

5. (Extra Credit) Below are 4 different computer plots for the parametric equation $\mathbf{r}(t) = \langle \cos(t), \sin(t) \rangle$ for values of t ranging from 0 to 2π, 200π, 2000π and 20000π respectively. Explain why we might get different pictures for each of the graphs.



Inaccuracies arise as the computer draws the figure by connecting points calculated as on the curve. A higher range means points get pulled further apart thus, the line segments get pulled "into" the circle

Please indicate the following:
I have a class before (after) Calc 3 on Thursdays.
I have a class before (after) Calc 3 on Fridays.