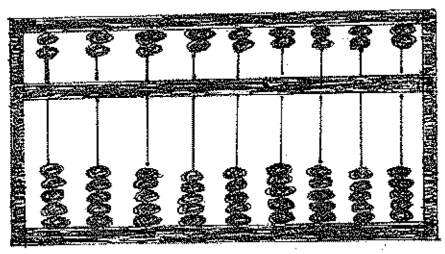
KEY

Math 225: Quiz the Fourth

February 16, 2012

This quiz is closed book and closed notes. You may use your calculator for the purposes of arithmetic and for plotting equations, if helpful. When asked for specific values, however, you must show the relevant algebra. READ ALL DIRECTIONS CAREFULLY. You have the remainder of the period.



APPLE'S "iReckon" NEVER CAUGHT ON.

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1. (a) Find the curvature, $\kappa(t)$, for $\mathbf{r}(t) = \langle 4\cos t, 4\sin t, 3t\rangle$

$r' = \langle -4\sin t, 4\cos t, 3\rangle \quad |r'| = 5$

$r'' = \langle -4\cos t, -4\sin t, 0\rangle$

$r' \times r'' = \langle 12\sin t, -12\cos t, 16\sin^2 t + 16\cos^2 t\rangle$

$|r' \times r''| = \sqrt{144+256} = \sqrt{400} = 20.$

$$K(t) = \frac{|r' \times r''|}{|r'|^3} = \frac{20}{125}$$

(b) Is curvature changing over time? Explain why or why not. (Hint: Think about the shape of the curve.)

No. The curve is a helix tracing upward at a constant rate, hence curvature stays constant.

(c) How would the curvature be different for $\mathbf{r}(t) = \langle 4\cos t, 4\sin t, 3t^2\rangle$. (Note: Do NOT try to calculate curvature directly here).

In this case, the rate of trace is increasing, thus curvature would <u>decrease</u> over time.

2. A particle moves with position function $\mathbf{r}(t) = \langle t \ln t, t, e^{-t} \rangle$.

(a) Find $\mathbf{v}(t)$

$$\vec{v}(t) = \vec{r}\,'(t) = \left\langle t\left(\tfrac{1}{t}\right) + 1(\ln t), 1, -e^{-t} \right\rangle$$
$$= \langle 1 + \ln t, 1, -e^{-t} \rangle$$

(b) Find $\mathbf{a}(t)$

$$\vec{a}(t) = \vec{r}\,''(t) = \vec{v}\,'(t)$$
$$= \left\langle \tfrac{1}{t}, 0, e^{-t} \right\rangle$$

3. Find the tangential and normal components of acceleration for $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + 3t\mathbf{k}$.

$$\vec{r}(t) = \langle t, t^2, 3t \rangle$$
$$\vec{r}\,'(t) = \langle 1, 2t, 3 \rangle$$
$$r''(t) = \langle 0, 2, 0 \rangle$$

$$a_T = \frac{r' \cdot r''}{|r'|} = \frac{4t}{\sqrt{10+4t^2}}$$

$$a_N = \frac{|r' \times r''|}{|r'|} = \frac{|\langle -6, 0, 2 \rangle|}{\sqrt{10+4t^2}} = \frac{\sqrt{40}}{\sqrt{10+4t^2}}$$

4. Identify the surface given by each equation. For each surface, also state which values of x are permissible.

(a) $32x^2 + 2y^2 + 72z^2 = 288$ $\rightarrow$ ellipsoid

$$\frac{x^2}{9} + \frac{y^2}{144} + \frac{z^2}{4} = 1 \qquad \underline{-3 \le x \le 3.}$$

(b) $3x = 27y^2 + 9z^2 + 15$

$x = 9y^2 + 3z^2 + 5$ elliptical paraboloid. $x \geq 5$

(c) $3x = 27y^2 - 9z^2 + 15$

$x = 9y^2 - 3z^2 + 5$ hyperbolic paraboloid. x takes on all values.

(d) $32x^2 - 2y^2 - 72z^2 = 288$ hyperboloid of 2 sheets

$$\frac{x^2}{9} = \frac{y^2}{144} + \frac{z^2}{4} + 1 \qquad \frac{x^2}{9} \geq 1$$

$x \geq 3$ or $x \leq -3$.

5. (Bonus) You may have 0.5 points extra credit or 1.5 points extra credit. (Note: If more than 25% of you take 1.5 points, no one gets any extra credit.)