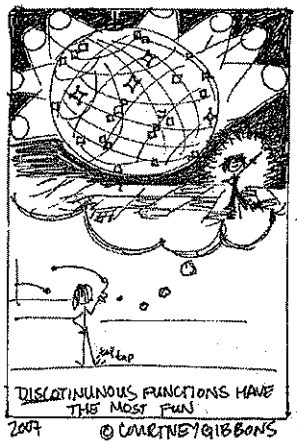
Math 225: Quiz the Fifth

KEY

This quiz is closed book and closed notes. You may use your calculator for the purposes of arithmetic and for plotting equations, if helpful. When asked for specific values, however, you must show the relevant algebra. READ ALL DIRECTIONS CAREFULLY. You have the remainder of the period.



1. Find the following limits, if they exist.

(a)

$$\lim_{(x,y)\to(0,0)} \frac{x}{\sqrt{x^2+y^2}}$$

If $x=0$, we have $\lim_{y\to 0} \frac{0}{\sqrt{y^2}} = 0$

If $y=0$ we have $\lim_{x\to 0} \frac{x}{\sqrt{x^2}} = 1$

(b)

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+16}-4}$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+16}-4} \cdot \frac{\sqrt{x^2+y^2+16}+4}{\sqrt{x^2+y^2+16}+4}$$

$$= \lim_{(x,y)\to(0,0)} \frac{(x^2+y^2)\left(\sqrt{x^2+y^2+16}+4\right)}{x^2+y^2+16-16} = \lim_{x,y\to 0,0} \sqrt{x^2+y^2+16}+4 = 8.$$

2. Consider the function $f(x,y) = x^3y + 2x + y^2$.

(a) Find f_x and f_y.

$f_x = 3x^2y + 2$

$f_y = x^3 + 2y$

(b) Find the equation to the tangent plane at $(3,2)$.

$f_x(3,2) = 3 \cdot 9 \cdot 2 + 2 = 56$

$f_y(3,2) = 27 + 4 = 31$

$f(3,2) = 27 \cdot 2 + 2 \cdot 3 + 4 = 64$

T_{Plane} $z = 64 + 56(x-3) + 31(y-2)$

(c) Use the tangent plane to approximate $f(3.1, 1.95)$. (no calculators, please!)

$f(3.1, 1.95) \approx 64 + 56(3.1-3) + 31(1.95-2)$

$= 64 + 5.6 - 1.55$

$= 68.05$

3. Let

$$z = \frac{x-y}{x+y}.$$

Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

$$\frac{\partial z}{\partial x} = \frac{(x+y)(1) - (x-y)(1)}{(x+y)^2} = \frac{2y}{(x+y)^2}$$

$$\frac{\partial z}{\partial y} = \frac{(x+y)(-1) - (x-y)(1)}{(x+y)^2} = \frac{-2x}{(x+y)^2}$$

4. Suppose that, for a function $f(x,y)$, we know that $f_x(x,y) = 3x^2 + \cos(y) + 2xy$. Give at least three possibilities for $f_y(x,y)$. Note: Your examples should differ by more than just a constant!

$f_x = 3x^2 + \cos y + 2xy$

$f = x^3 + x\cos y + x^2 y + g(y)$

$f_y = 0 + x(-\sin y) + x^2 + g'(y)$

Possibilities $f_y = -x\sin y + x^2 + 6y$

or $f_y = -x\sin y + x^2 + e^y$

or $f_y = -x\sin y + x^2 + \arccos y$

5. Extra Credit:

(a) What is your birthday? (Month and Day only. No years. Please.)

(b) Of the 24 of us (myself included), what is the approximate probability that at least 2 of us have the same birthday?

i. 8 % ii. 16 % iii. 32 % iv. 50 %