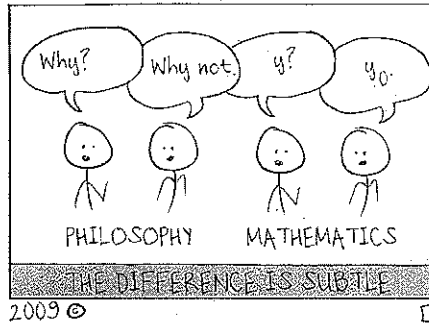


KEY

Math 225: Quiz the Sixth

This quiz is closed book and closed notes. You may use your calculator for the purposes of arithmetic and computation only. When asked for specific values, however, you must show the relevant algebra. You have the remainder of period.



1. Calculate $f_{xy}(x, y)$ for the function $f(x, y) = \arcsin(x^2) + 3 \tan^2(y) + 2xy$

$$\begin{aligned} f_{xy} &= \frac{\partial}{\partial y} \left(\frac{\partial}{\partial x} (\arcsin(x^2) + 3 \tan^2 y + 2xy) \right) \\ &= \frac{\partial}{\partial y} \left(\frac{\partial}{\partial x} \arcsin x^2 + 0 + 2y \right) \\ &= \frac{\partial}{\partial y} (2y) = 2 \end{aligned}$$

(3)

125
P

$$402 + 80 + 16 + 3$$

501

2. Let $f(x, y) = x^3 - x^2y + 6y^2$

(a) Find ∇f at $(2, 1)$

$$\vec{\nabla} f = \langle 3x^2 - 2xy, -x^2 + 12y \rangle$$

$$\begin{aligned}\vec{\nabla} f_{(2,1)} &= \langle 12 - 4, -4 + 12 \rangle \\ &= \langle 8, 8 \rangle\end{aligned}$$

(b) Find $D_{\mathbf{u}}f$ as we move from $(2, 1)$ towards $(5, -3)$.

$$(2, 1) \text{ to } (5, -3) \rightarrow \langle 3, -4 \rangle \xrightarrow{\vec{u}} \left\langle \frac{3}{5}, -\frac{4}{5} \right\rangle$$

$$D_{\mathbf{u}} f = \langle 8, 8 \rangle \cdot \left\langle \frac{3}{5}, -\frac{4}{5} \right\rangle = \frac{-8}{5}$$

(c) Find $D_{\mathbf{u}}f$ at $(2, 1)$ ~~towards~~ in the direction of $(1, 1)$. $\vec{u} \rightarrow \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$

$$D_{\mathbf{u}} f = \langle 8, 8 \rangle \cdot \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle = \frac{16}{\sqrt{2}}$$

3. Let $xyz + x^2 + xy + xz = 2$.

(a) Find the equation of the tangent plane to this surface at the point $(-1, 2, -1)$.

$$F = xyz + x^2 + xy + xz = 2$$

$$\vec{n} = \nabla F = \langle yz + 2x + y + z, xz + x, xy + x \rangle \Big|_{(-1, 2, -1)}$$

$$= \langle -2 - 2 + 2 - 1, 1 - 1, -2 - 1 \rangle$$

$$\langle -3, 0, -3 \rangle$$

$$\text{Plane: } -3(x+1) - 0(y-2) - 3(z+1) = 0$$

(b) For each partial derivative listed below, either calculate at $(-1, 2, -1)$ it or state why it is not defined. (Use your gradient from above to help save you some work here).

i. $\frac{\partial z}{\partial x}$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(-3)}{-3} = -1$$

ii. $\frac{\partial y}{\partial x}$

$$\frac{\partial y}{\partial x} = -\frac{F_x}{F_y} = \frac{-(-3)}{0} \text{ undefined}$$

iii. $\frac{\partial x}{\partial y}$

$$\frac{\partial x}{\partial y} = -\frac{F_y}{F_x} = -\frac{0}{(-3)} = 0$$

4. Wheat production (W) is dependent on temperature (T) and rainfall (R). It is estimated that temperature is increasing at a rate of $0.05^\circ \text{C}/\text{year}$ and rainfall is decreasing at a rate of $0.1 \text{ cm}/\text{year}$. It is also estimated that, at current levels of production, $\frac{\partial W}{\partial T} = -1.5$ and $\frac{\partial W}{\partial R} = 4$.

(a) Give a physical description of the significance of the signs of the partial derivatives $\frac{\partial W}{\partial T}$ and $\frac{\partial W}{\partial R}$.

$\frac{\partial W}{\partial T} < 0$ As Temp increases, wheat production decreases

$\frac{\partial W}{\partial R} > 0$ As Rainfall increases, wheat production increases

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(b) Using these estimates, find $\frac{dW}{dt}$.

$$\frac{dW}{dt} = \frac{\partial W}{\partial R} \frac{dR}{dt} + \frac{\partial W}{\partial T} \frac{dT}{dt}$$



$$\begin{aligned} \frac{dW}{dt} &= 4(-0.1) + (-1.5)(0.05) \\ &= -0.4 - 0.075 \\ &= -0.475 \text{ bushels/yr.} \end{aligned}$$

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5. (Extra Credit) You may take either one half-point for yourself, or zero points for yourself and give one-tenth of a point to everyone else in the class. Your extra credit score will be the total of points taken for yourself and given to you by others.

I'll kick in .1 for the class