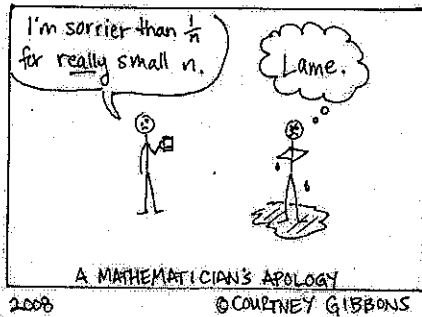


# KEY

## Math 225: Quiz the Seventh March 29, 2012

This quiz is closed book and closed notes. Please justify all of your answers. You have 40 minutes.



1. For each pair of partial derivatives below, find a function  $f(x, y)$  that satisfies the derivatives, or explain why no such function exists.

(a)  $f_x = 2x + y^2$      $f_y = 2y + y^2$   
 $f_{xy} = 2y$      $f_{yx} = 0$     no fun exists.

(b)  $f_x = 2x + 2xy$      $f_y = 2xy + y^2$   
 $f_{xy} = 2x$      $f_{yx} = 2y$     no fun exists

2. Find and classify the critical points of

$$f(x, y) = x^3 - 3x + \frac{y^2}{2} + y.$$

$$f_x = 3x^2 - 3 = 0 \quad 3(x^2 - 1) = 0 \quad x = \pm 1$$

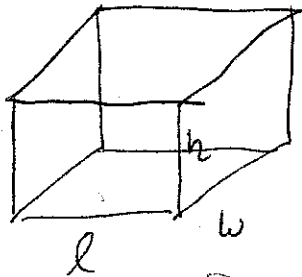
$$f_y = y + 1 = 0 \quad y + 1 = 0 \quad y = -1$$

CP's  $(1, -1)$      $(-1, -1)$

$$D = f_{xx} f_{yy} - (f_{xy})^2 = (6x)(1) - 0 = 6x \quad (f_{xx} > 0)$$

$D(1, -1) = 6 > 0$  has a local min @  $(1, -1)$   
 $D(-1, -1) = -6 < 0$  has a saddle point @  $(-1, -1)$

3. A box without a top is to be constructed of  $100 \text{ cm}^2$  of material. Find the box with the maximum volume.



STD Method gets ugly, fast! \*

$$SA = lw + 2lh + 2wh = 100$$

$$V = lwh$$

$$h = \frac{100 - lw}{2l + 2w}$$

$$h = \frac{100 - l^2}{4l}$$

$$V = lw \left( \frac{100 - lw}{2l + 2w} \right)$$

$$(l+w)(100 - 2lw) = (100 - l^2)(l) \\ 100l + 100w - 2l^2w - 2lw^2 = 100l - l^3$$

$$l = w = \frac{h}{2}$$

$$h =$$

$$\begin{aligned} lw &= (2l+2w)h & 2lh &= lw \\ lh &= (l+2h)w & 2h &= w=l \\ kh &= (w+2h)l \end{aligned}$$

$$\begin{aligned} lw + 2lh + 2wh &= 100 \\ 5l^2 &= 100 \\ l^2 &= 20 \\ l = w = \sqrt{20} &= 2\sqrt{5} \\ h &= \sqrt{5} \end{aligned}$$

$$V = 10\sqrt{5}$$

4. Find the point on the plane  $x + 2y + 3z = 6$  that is closest to the origin. (Use whichever method you like here).

use Lagrange

$$D_{xy} = x^2 + y^2 + z^2$$

$$s.t. \quad x + 2y + 3z = 6$$

$$\begin{aligned} 2x &= \lambda \\ 2y &= 2\lambda \\ 2z &= 3\lambda \end{aligned}$$

$$\begin{aligned} x &= \frac{\lambda}{2} & y &= \frac{\lambda}{2} & z &= \frac{3\lambda}{2} \\ &= \frac{3}{7} & &= \frac{6}{7} & &= \frac{9}{7} \end{aligned}$$

$$x + 2y + 3z = 6$$

$$\frac{\lambda}{2} + 2\lambda + \frac{9\lambda}{2} = 6$$

$$\left( \frac{3}{7}, \frac{6}{7}, \frac{9}{7} \right)$$

$$\begin{aligned} \frac{14\lambda}{2} &= 6 \\ \lambda &= \frac{12}{14} = \frac{6}{7} \end{aligned}$$