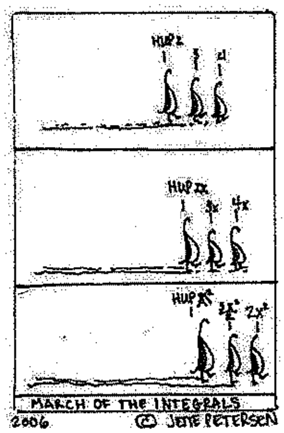
Math 225: Quiz the Eighth
April 5, 2012

This quiz is closed book and closed notes. Please justify all of your answers. You have 40 minutes.



1. Find the volume in the first octant bound by $x = 1$, $y = 2$, and $z = 4 - x^2 - y^2$

$$\int_0^1 \int_0^2 4 - x^2 - y^2 \, dy \, dx$$

$$= \int_0^1 4y - x^2 y - \frac{y^3}{3} \Big]_0^2 \, dx$$

$$= \int_0^1 8 - 2x^2 - \frac{8}{3} \, dx$$

$$= \int_0^1 \frac{16}{3} - 2x^2 \, dx$$

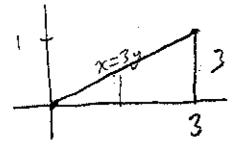
$$= \frac{16}{3}x - \frac{2}{3}x^3 \Big]_0^1$$

$$= \boxed{\frac{14}{3}}$$

2. Find

Sorry!

$$\int_0^1\int_{3y}^{3} e^{x^2}\,dx\,dy$$



$$= \int_0^1 \int_0^{\frac{x}{3}} e^{x^2}\,dy\,dx$$

$$= \int_0^1 \frac{x}{3} e^{x^2}\,dx$$

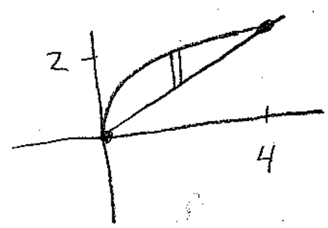
$$= \frac{1}{3}\left[\frac{e^{x^2}}{2}\right]_0^1$$

$$= \frac{e}{6} - \frac{1}{6} = \boxed{\frac{e-1}{6}}$$

3. Find

$$\iint_R x + y\,dA$$

where R is bound by $y = \frac{x}{2}$ and $y = \sqrt{x}$.



$$\int_0^4 \int_{\frac{x}{2}}^{\sqrt{x}} x+y\,dy\,dx$$

$$= \int_0^4 \left(xy + \frac{y^2}{2}\right)\Bigg|_{\frac{x}{2}}^{\sqrt{x}} dx$$

$$= \int_0^4 \left(x\sqrt{x} + \frac{x}{2}\right) - \left[\frac{x^2}{2} + \frac{x^2}{8}\right] dx$$

$$= \int_0^4 x^{3/2} + \frac{1}{2}x - \frac{5}{8}x^2\,dx = \frac{2}{5}x^{5/2} + \frac{x^2}{4} - \frac{5}{24}x^3\Bigg|_0^4$$

$$= \frac{64}{5} + \frac{16}{4} - \frac{40}{6} = \frac{64}{5} + 4 - \frac{20}{3}$$

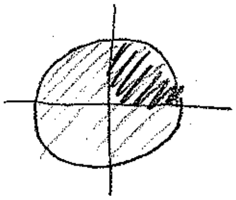
$$= \cancel{\frac{4}{5}} = \frac{192 + 60 - 100}{15}$$

$$= 152/15$$

4. Find

$$\iint_R xy\, dA$$

where R is bound by $x^2 + y^2 = 1$, $x = 0$ and $y = 0$.



$$\int_0^{\frac{\pi}{2}} \int_0^1 r\cos\theta \; r\sin\theta \; r\, dr\, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{r^4}{4} \cos\theta \sin\theta \Big|_0^1 \, d\theta$$

$$= \frac{1}{4} \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta \qquad \begin{array}{l} u = \sin\theta \\ du = \cos\theta\, d\theta \end{array}$$

$$= \frac{1}{8} \sin^2\theta \Big|_0^{\pi/2} = \frac{1}{8}$$

5. Find the area in the top half of the cardioid $r = 1 + \sin(\theta)$

$$\iint_R 1\, dA = \int_0^{\pi} \int_0^{1+\sin\theta} r\, dr\, d\theta$$

$$= \int_0^{\pi} \frac{(1+\sin\theta)^2}{2} \, d\theta$$

$$= \frac{1}{2} \int_0^{\pi} 1 + 2\sin\theta + \sin^2\theta \, d\theta = \frac{1}{2} \int_0^{\pi} 1 + 2\sin\theta + \left(\frac{1-\cos 2\theta}{2}\right) d\theta$$

$$= \frac{1}{2} \left[\frac{3}{2}\theta - 2\cos\theta - \frac{\cos 2\theta}{4} \right]_0^{\pi}$$

$$= \frac{1}{2} \left[\left(\frac{3\pi}{2} + 2 - \frac{1}{4} \right) - \left(0 - 2 - \frac{1}{4} \right) \right]$$

$$= \frac{1}{2} \left[\frac{3\pi}{2} + 4 \right] = \boxed{\frac{3\pi}{4} + 2}$$

6. (Bonus) Pretend to toss a coin five times. Record the outcomes as a list of heads and tails.

H, T, H, H, T