Math 225: Quiz Zero KEY

SPRING 2014

These problems are representative of the type that you have encountered in your mathematical past, and give some of the concepts that you'll be responsible for as we move through the new material. This will not be collected, though I will provide a solutions key online.

1. Give the domain and range for the function $f(x) = \ln(1 - x^2)$.

$f(x) = \ln x$ is only defined when $x > 0$

Thus, $f(x) = \ln(1-x^2)$ is only defined when

$1 - x^2 > 0$

or $1 > x^2$, or $1 > x > -1$

so $x \in (-1, 1)$

2. Find

$$\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 - 5x + 6}$$

Direct plugging gives $\frac{2^2 - 3\cdot 2 + 2}{2^2 - 5\cdot 2 + 6} = \frac{0}{0}$

So we need a bit of algebra: $\frac{x^2 - 3x + 2}{x^2 - 5x + 6} = \frac{(x-2)(x-1)}{(x-2)(x-3)}$

so $\lim_{x\to 2} \frac{x^2-3x+2}{x^2-5x+6} = \lim_{x\to 2} \frac{x-1}{x-3} = \frac{1}{-1} = \underline{-1}$

3. Find the maximum and minimum values of $f(x) = 3x - x^2$ on the interval $[1, 4]$

Note: Since $[1,4]$ is closed, f will achieve its min and max

Crit points $f'(x) = 3 - 2x$, $f'(x) = 0$ when $x = 3/2$

$f'' = -2 < 0$ so f has a local max @ $x = 3/2$

Testing Endpoints: $f(1) = 3 - 1 = 2$ $\quad f(3/2) = \frac{9}{2} - \frac{9}{4} = \frac{9}{4}$ $\quad f(4) = 3\cdot 4 - 4^2 = -4$

$f(3/2)$: MAX $\quad$ $f(4)$: MIN

4. Let $f(x) = \sqrt[3]{x}$. Find the tangent line to $f(x)$ at $x = 8$ and use it to approximate $\sqrt[3]{8.02}$.

$f(x) = \sqrt[3]{x} = x^{1/3}$ $\quad f(8) = 2$

$f'(x) = \frac{1}{3} x^{-2/3}$ $\quad f'(8) = \frac{1}{3}(8)^{-2/3} = \frac{1}{12}$

Line: $y - 2 = \frac{1}{12}(x - 8)$

$\sqrt[3]{8.02} \approx 2 + \frac{1}{12}(8.02 - 8) = 2 + \frac{.02}{12} = 2.001\overline{666}$

5. (a) Find

$$\int_0^{0.5} \frac{t\,dt}{\sqrt{1-t^4}}$$

Let $u = t^2$

Then $du = 2t\,dt$

lower lim = 0

upper lim = 0.25

$$\int_0^{0.25} \frac{\frac{1}{2}\,du}{\sqrt{1-u^2}} = \frac{1}{2}\arcsin u \Big]_0^{0.25}$$

$$= \frac{1}{2}\arcsin\frac{1}{4}.$$

(b) Discuss

$$\int_0^1 \frac{t\,dt}{\sqrt{1-t^4}}$$

Note: This integral is ~~improper~~ as $\frac{t}{\sqrt{1-t^4}}$ is undefined @ $t=1$.

Instead, we investigate

$$\lim_{b\to 1} \int_0^b \frac{t}{\sqrt{1-t^4}}\,dt = \lim_{b\to 1} \frac{1}{2}\arcsin t^2 \Big]_0^b$$

$$= \frac{1}{2}\,\frac{\pi}{2} = \boxed{\frac{\pi}{4}}$$

6. Find

$$\int \cos^2\theta\,d\theta$$

We need a TRIG IDENTITY

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

So $\int \cos^2\theta\,d\theta = \int \frac{1+\cos 2\theta}{2}\,d\theta$

$$= \frac{\theta}{2} + \frac{\sin 2\theta}{4} + C$$

7. Find both $\frac{d}{dx}xe^x$ and $\int xe^x\,dx$

$$\frac{d}{dx}\,x e^x = e^x + xe^x$$

PROD RULE

$$\int x e^x\,dx = xe^x - \int e^x\,dx = \underline{xe^x - e^x + C}$$

IBP with $u = x$, $dv = e^x dx$, $du = dx$, $v = e^x$